Thermodynamically Consistent Decoupled Shear-Volumetric Strain Model and CTH Implementation

A.D. Resnyansky

Weapons Systems Division
Defence Science and Technology Organisation

DSTO-TR-2299

ABSTRACT

Many hydrocodes, such as LS-DYNA and CTH, require the decoupling of the shear from volumetric response in a material model used. A constitutive model is formulated, which decouples the responses of a rate sensitive material. Basis of the model is a general Maxwell-type viscoelastic model, which, however, is not originally decoupled and, thus, not suitable for implementation in the hydrocodes. The formulation provides the thermodynamic consistency for the case of small deviatoric elastic deformations and unrestricted volumetric response. A schematic of implementation in CTH is briefly described. Numerical illustrations demonstrate agreement of the CTH calculations with calculations available in the literature.

RELEASE LIMITATION

Approved for public release
Thermodynamically Consistent Decoupled Shear-Volumetric Strain Model and CTH Implementation

Executive Summary

The development of advanced weaponry demands science and industry pay attention to novel energetic materials and materials with enhanced protective properties, including composite materials, porous and multi-phase mitigants, advanced materials manufactured with nanotechnology, etc. The response of these materials needs to be predicted and this raises new challenges for the area of computer modelling of the material response to extreme pressure and temperature. Advanced shock physics codes are a solid basis for modelling efforts. Among them hydrocodes are the most relevant to the level of loads and temperatures because they are able to deal both with the material behaviour in conditions of hydrodynamic flow and with the elastic-plastic deformation at moderate loads when the material strength is important. Hydrodynamic (strength negligible) flows due to extreme loads are typical for materials subject to the hyper-velocity impact by shaped charge jets and for materials in direct contact with detonating energetic materials. The effect of these conditions on enhanced materials requires consideration of multi-phase behaviour with phase transitions, however, such models are yet to be developed. The elastic-plastic (strength relevant) deformations prevail in materials at the load levels typical of high-velocity fragmentation impact or of blast effects from charges in a certain proximity to the target. The elastic-plastic response under these conditions is of the constitutive type, i.e., the materials manifest rate sensitivity and require advanced modelling approaches. In summary, the development and use of advanced models in commercially available hydrocodes is a great challenge.

The modelling capability in DSTO is supported by several hydrocodes. The LS-DYNA 3D hydrocode (originally a Lagrangian code) has been employed for a number of years in DSTO. It has proved its efficiency for modelling the weapon and terminal effects in problems involving moderate deformations and requiring a good resolution of contact material interfaces. A user-defined material model is relatively simple to incorporate within the DYNA interface though no well-documented implementation procedures are established. A number of material models have been implemented in this code in DSTO exploring the constitutive model capabilities for:

i) conventional materials subject to high-velocity fragment impact [1] and materials involved in the shaped charge jet formation, when studying the weapon effects of the two-stage follow-through grenade weapon [2],

ii) composite materials subject to ballistic impact for the Army Reconnaissance Helicopter project [3],

iii) concretes subject to hyper-velocity impact, when studying the target effects against the multi-stage weapon threats [2, 4], and

iv) materials manifesting multi-phase features, when simulating the underwater explosion in a Navy project [5].
However, for the problems such as counter-IED (Improvised Explosive Devices) dealing with large deformations, intensive flows and high multi-phase/ multi-material mixing a Eulerian code is more suitable. Therefore, the present work is attempting to gather the implementation experience for the CTH hydrocode developed by Sandia National Laboratories in the US. Implementation procedures for the code are quite well documented. The implementation difficulties are mainly associated with the necessity for the user to intervene in a number of entry points of the code, the number of which and the places of entry depend on the nature of the model to be incorporated. Due to the release features of the code not all necessary points of entry may be easily accessible, this is another challenge for the user.

The present work adapts a rate sensitive strength model for conventional materials, which has been published earlier in the literature, to a form decoupling the shear and volumetric responses. The present formulation is in agreement with the CTH implementation requirements and it preserves the thermodynamic consistency of the model. In addition, this formulation suggests a general thermodynamically consistent way of decoupling between the shear and volumetric responses of a material, which might be useful for the development of advanced strength models requiring this decoupling. The model along with its implementation is tested with a number of impact and shaped charge problems. The CTH calculations are compared with numerical solutions and experiments available in the literature and a good agreement is observed. Thus, the present work establishes the model implementation capability in DSTO for the CTH hydrocode.

References


A. D. Resnyansky
Weapons Systems Division

Anatoly Resnyansky obtained a MSc in Applied Mathematics and Mechanics from Novosibirsk State University (Russia) in 1979. In 1979-1995 he worked in the Lavrentyev Institute of Hydrodynamics (Russian Academy of Science) in the area of constitutive modelling for problems of high-velocity impact. Anatoly obtained a PhD in Physics and Mathematics from the Institute of Hydrodynamics in 1985. In 1996-1998 he worked in a private industry in Australia. He joined the Weapons Effects Group of the Weapons Systems Division (DSTO) in 1998. His current research interests include constitutive modelling and material characterisation at high strain rates, ballistic testing and simulation, and theoretical and experimental analysis of multi-phase flows. He has more than eighty papers published internationally in this area.
Contents

1. INTRODUCTION ............................................................................................................. 1

2. CONSTITUTIVE MODEL ................................................................................................. 2

3. CONSTITUTIVE RELATIONS AND EOS .................................................................. 5

4. THERMODYNAMIC CORRECTNESS ....................................................................... 6

5. CTH IMPLEMENTATION ................................................................................................. 9
   5.1 Input Block ............................................................................................................. 9
      5.1.1 UINEP modifications .................................................................................... 9
      5.1.2 UINCHK modifications .............................................................................. 10
      5.1.3 UINISV modifications ............................................................................... 10
   5.2 Lagrangian Block ..................................................................................................... 10
      5.2.1 ELSG modifications .................................................................................... 11
   5.3 Eulerian Remap Block ......................................................................................... 11
      5.3.1 EREB modifications .................................................................................... 11

6. MODELLING OF SHOCK BEHAVIOUR ................................................................. 12

7. BALLISTIC IMPACT AND TAYLOR TEST MODELLING ....................................... 13

8. SHAPED CHARGE JET MODELLING ........................................................................ 17

9. HYPER-VELOCITY IMPACT MODELLING ............................................................... 19

10. CONCLUSIONS ............................................................................................................ 28

ACKNOWLEDGEMENTS ................................................................................................. 28

REFERENCES .................................................................................................................... 29
1. Introduction

A large number of advanced material models describing the strength effects have been developed in the last decades. Some of these models (hyperelastic material models) that follow the thermodynamic approach employ a single thermodynamic potential that enables one to derive all thermodynamic dependent variables such as pressure or stresses from the potential. For many of the models obtained with this methodology, it is hard to separate one type of response from another, such as the deviatoric response (managed mainly by the shear stresses) from the volumetric/bulk response (managed by hydrostatic pressure). An alternative type of models follows the decoupling approach directly, thus, ignoring possible thermodynamic restrictions. These models concentrate mainly on empirical description of the constitutive processes in material. It should be noted that the separation is rather conventional [1]; it may have some physical grounds if certain physical mechanisms of the material behaviour might be referred to certain process timeframe or load level for one type of response, and the other mechanisms out of that timeframe or load are responsible for the alternative type of response. Whereas, the general potential-based way is attractive from the thermodynamics viewpoint, serious difficulties are met when implementing the corresponding models in conventional commercial hydrocodes such as LS-DYNA [2] and CTH [3]. On the other side, the majority of the decoupled models, as mentioned before, which are easy to incorporate and which follow the implementation requirements closely, may not be consistent with the thermodynamics requirements. This may result in troubles that are associated with the transition from one type of response into another, such as the situation where both the shear and volumetric responses (material strength and hydrodynamic pressure) are critical for analysis.

The present work proposes a modification of a known rate sensitive model first reported on for the case of finite elastic deformations in [4] and considered in detail in [5]. The model has been used for calculation of shock propagation [6] and high-velocity [7] and jet-formation problems [8], using the model in the finite deformation formulation, for which a special hydrocode was designed [7]. The basic elements of the model have been redesigned for a DYNA implementation suitable for small elastic deformations [9]; however, the thermodynamic correctness of the model implementation [9] was disregarded. The present modification of the model [4] for the case of small elastic shear deformations and unrestricted volumetric response (within the thermodynamics limitations) focuses on 1) decoupling of the model into the form that is in agreement with the hydrocode implementation requirements; and 2) the thermodynamic correctness of the model. It should be noted that the present decoupling cannot be extended to the situations and materials where elastic shear deformations are large (such as the general non-linear elasticity). However, whenever the energy potential coefficients (elastic modulus) are dependent on the characteristics of the volumetric response such as pressure or density, there are no restrictions on the amount of volumetric deformation in the model.

The model decoupling is associated with two major model elements relevant to the hydrocode implementation: 1) Constitutive Equations (CE) describing evolution of deviatoric elastic deformations directly linked with shear stresses; and 2) an Equation Of State (EOS) describing the material’s volumetric (bulk) response. In physical reality, material does not distinguish
one form of the response from the other, but the model formalism allows us to separate the
deviatoric part of the response from the spherical one. Thermodynamically, however, these
responses are usually treated differently; in fact, they are subject to the compatibility between
the parameters responsible for each of the responses. The present approach is an attempt to
combine both responses into a single thermodynamic consideration. For the case of small
shear deformations, this combination appears to be possible due to the fact that the
generalized deviatoric forces and displacements are in a straightforward link for this case.

The CTH implementation is briefly described with references to the code structure available in
public literature. The main CTH structure includes the Input Block (IB), Lagrangian Block
(LB), and Eulerian Remap Block (ERB) [10]. Publications [11, 12] describe the code structure in
slightly more details with reference to specific routines of the code. The present
implementation includes modifications to i) the input subroutines of the Elasto-Plastic (EP)
section in the IB part, ii) evaluation of deviatoric stresses in the LB part; and iii) recalculation
of pressure, energy, and temperature in accordance with EOS in the ERB part. This experience
gives us a way to implement a wide variety of models within the model framework
employing a single set of conservation laws. A recent DSTO work on formulation and
development of multi-component and multi-phase models within this framework allows us to
expand this modelling approach over the material range covering porous, brittle and multi-
phase materials [13-18]. The model implementation is tested with a number of well-known
problem set-ups compared with calculations conducted with the model [4, 5] including: one-
dimensional shock propagation problems, the results for which are compared with the
calculations [6]; Taylor test problem; high-velocity impact problem results compared with the
calculations [7, 9]; Shaped Charge (SC) jet formation results compared with the calculations
and experiments [19]; and hypervelocity impact results compared with similar calculations
[7]. The calculations demonstrate a good agreement of the CTH calculations with the literature
data and prove that the constitutive effects are well traced with the use of the CTH numerical
scheme.

2. Constitutive Model

A thermodynamically consistent formulation of the Maxwell-type viscoelastic model has been
published in [4, 5]. The system was formulated for the case of finite deformations represented
by the metric tensor of finite deformations, which is a combination of components of the
gradient deformation tensor producing the symmetric tensor. It should be noted that the
system can be written down in the form of conservation laws only for the gradient
deformation tensor [5] but not for the metric tensor of finite deformations. The Almansi tensor
of finite deformations $\varepsilon_{ij}$ is frequently used as a strain measure because it is zero in the absence
of deformations, which is convenient when approximating small deformations. The Almansi
strain tensor is a simple combination of the metric tensor [5]. The Maxwell-type viscoelastic
model, which is reduced to the case of small elastic deformations, takes the following form [4,
5]:
The equations are written down in the Eulerian coordinate system, where \(\frac{d}{dt} = \frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x_k}\) denotes the substantial derivative. Here, \(\varepsilon_{ij}\) and \(\sigma_{ij}\) are components of the tensor of small elastic deformations and the stress tensor, \(u_i\) are velocity components, and \(\rho\) and \(E\) are density and specific internal energy. The prime sign refers the deviatoric part of a tensor, so \(\varepsilon_{ij}' = \varepsilon_{ij} - \delta_{ij} \frac{\varepsilon_{kk}}{3}\), where \(\delta_{ij}\) are components of the unit tensor. Definitions of small elastic deformations are usually indistinguishable. For example, the formulation (1) is identical to the model's system written down for the small logarithmic elastic deformations [20].

When analysing continuous flows, the continuity equation (mass conservation law) should normally be satisfied:

\[
\frac{d\rho}{dt} + \rho \frac{\partial u_k}{\partial x_k} = 0. \tag{2}
\]

However, for any elasto-plasticity theory the conservation law (2) is simply a compatibility equation (e.g., see [4, 5]). That is, this equation is merely a consequence of the constitutive equations from (1) because density is changing only due to the change of elastic deformations. Pre-given functions \(\tau(\varepsilon_{ij}, T)\) and \(E(\varepsilon_{ij}, S)\), where \(T\) and \(S\) are temperature and specific entropy, are closing the model's system of equations (1). The function \(\tau(\varepsilon_{ij}, T)\) controlling the shear response is obtained from the experimental data on the yield limit versus strain rate (e.g., see [9] on how this can be done) and the potential \(E(\varepsilon_{ij}, S)\) (specific internal energy) controlling the volumetric response is obtained from a variety of high-pressure response data such as Hugoniot, etc (e.g., see [21]). For the case of small elastic deformations the second thermodynamic law takes the following form (e.g., see [17]):

\[
T \, ds = dU - \sigma_{ij} \, dv_{ij}, \tag{3}
\]

where \(s = \rho S\) is the entropy and \(U = \rho E\) is the internal energy. The summation in (3) is conducted over all \(i\) and \(j\) combinations. It should be kept in mind that the density change is ignored in (3) for the case of small elastic deformation. This consideration is completely coupled and calculation of stresses and temperature from the energy potential is conducted from (3) as follows

\[
T = \frac{\partial U}{\partial s}, \quad \sigma_{ij} = \frac{\partial U}{\partial \varepsilon_{ij}}. \tag{4}
\]

However, when the shear strains are small and the volume change is significant, the formulation (3) is of limited use. Therefore, we suggest a formulation that decouples the shear from bulk responses, allowing one to consider the shear strains to be small and the density change unlimited (within the thermodynamics limitations).

To conduct the decoupling we will assume for a moment that the density change is negligible during the strength related processes that are governed by the shear stresses in the
constitutive equation of (1). We focus on the deviatoric components of the tensor of elastic deformation that gives the following constitutive equations using (1):

\[
\frac{d\varepsilon_{ij}'}{dt} - \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} : \varepsilon_{ij} \right) = -\frac{\varepsilon_{ij}'}{\tau}.
\]  

We introduce the following 'density-normalized' deviatoric elastic deformations \( e_{ij} = \varepsilon_{ij}' / \rho \). Then, with the assumption of the negligible change for density when varying \( e_{ij} \), the constitutive equation (5) can be rewritten as follows:

\[
\frac{de_{ij}}{dt} - \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} : \varepsilon_{ij} \right) = -\frac{\rho e_{ij}}{\tau}.
\]  

Now, we can tabulate this equation to be the one describing the shear response. The functions of the constitutive equations could be fitted to the experimental data similarly to the procedure for the constitutive equations in (1). It is obvious that the following restrictions are imposed on the tensor \( e_{ij} \):

\[
e_{33} = -e_{11} - e_{22}, \quad e_{12} = e_{21}, \quad e_{23} = e_{32}, \quad e_{31} = e_{13}.
\]  

Therefore, only 5 of 9 equations of (6) should really be calculated. However, from the system (6) density cannot be found, so the continuity equation (2) should be used, too. It is interesting that the system (6) can be formulated in the conservation law form. To complete the decoupling we have to introduce the deviatoric part of the stress tensor \( s_{ij} \) and the spherical (negative) part or pressure \( p \):

\[
s_{ij} = \sigma_{ij} + p \delta_{ij}, \quad p = -\sigma_{kk}/3.
\]  

Then, the complete system of equations for this model can be written down in the conservation laws form as follows:

\[
\begin{align*}
\frac{\partial \rho e_{ij}}{\partial t} + \frac{\partial \rho u_i e_{ij}}{\partial x_k} &= -\frac{\partial \rho u_i}{\partial x_j} - \frac{\partial \rho u_j}{\partial x_i} + \frac{\partial \rho u_k}{\partial x_k} : \varepsilon_{ij} = -\frac{\rho e_{ij}}{\tau}, \\
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_k}{\partial x_k} &= 0, \\
\frac{\partial \rho \left( E + u^2/2 \right)}{\partial t} + \frac{\partial \left( \rho u_j \left( E + u^2/2 \right) - s_{ij} u_i + p u_j \right)}{\partial x_j} &= 0.
\end{align*}
\]  

Here, \( u^2 = u_i : u_i \) is square of the velocity module. The thermodynamic identity (3) takes the following form for the present case:
\[ T\, dS = dE - s_{ij}\, de_{ji} + p\, dV, \quad (10) \]

where \( V = 1/\rho \) is the specific volume. Thus, we have 11 independent thermodynamical variables, namely, \( e_{ij}, \rho, \) and \( S \) and 11 dependent variables: \( s_{ij}, p, \) and \( T. \) The dependent variables are calculated from the identity (10) as follows:

\[ T = \frac{\partial E}{\partial S}, \quad p = \rho^2 \frac{\partial E}{\partial \rho}, \quad s_{ij} = \frac{\partial E}{\partial e_{ji}}. \quad (11) \]

As mentioned earlier, from (7) the number of independent variables to be calculated is reduced to 7 (so, only the 7 are independent) and the number of dependent variables calculated in (10) is correspondingly reduced as a consequence of (7), resulting in

\[ s_{33} = -s_{11} - s_{22}, \quad s_{12} = s_{21}, \quad s_{23} = s_{32}, \quad s_{31} = s_{13}. \quad (12) \]

However, the conditions (7) and (12) should not be invoked in manipulations with the thermodynamic relations. Therefore, these conditions are used only at the calculation stage when the closing relations (the consequences of the thermodynamics manipulations) have been obtained. As a result, the number of equations in (9) to compute is properly reduced.

Thus, the system (9) along with a pre-given energy potential (EOS) \( E(\rho, e_{ij}, S) \) and the constitutive function \( \tau(\rho, e_{ij}, S) \) generate a model describing behaviour of a material with small elastic shear deformations. However, the bulk response is unrestricted providing that the thermodynamic correctness is satisfied. The EOS and constitutive dependences are specified below.

### 3. Constitutive Relations and EOS

In the present work we choose a reduced form of the Mie-Gruneisen-type EOS developed in [21, 22] and used in [9, 13, 18]:

\[ E(\rho, D, S) = \frac{a_0^2}{2\alpha_0} \left( \delta^{\alpha_0} - 1 \right)^2 + 2b_0^2 \delta^{\beta_0} D + c_0 T_0 \delta^{\gamma_0} \left[ \exp \left( \frac{S}{c_v} \right) - 1 \right]. \quad (13) \]

Here, \( a_0, b_0, \) and \( \gamma_0 \) (Gruneisen coefficient) are material constants, \( c_v \) is the thermal capacity, and \( \delta = \rho/\rho_0 \) with \( \rho_0 \) to be initial density. The constant \( a_0 \) is bulk sound velocity that is linked with the longitudinal and shear sound velocities \( c_0 \) and \( b_0 \) as follows

\[ a_0^2 = c_0^2 - 4b_0^2/3, \quad (14) \]

and \( D \) is the second invariant of the strain deviator:

\[ D = e_{ij} \cdot e_{ji}/2 = \rho^2 e_{ij} \cdot e_{ji}/2 = \rho^2 d, \quad d = e_{ij} \cdot e_{ji}/2. \]

Thus, the energy potential takes the following form in the independent variables:
Shear stresses are calculated from (11) with use of (15) as

\[
\tau_{ij} = E_{ij} = E_{ij} = E_{ij} = 2\rho b^2\delta_{\delta_j} e_{ij} = 2\rho b^2\delta_{\delta_j} \rho e_{ij} = 2\rho b^2\delta_{\delta_j} \epsilon_{ij}.
\]  

The proportionality coefficient between shear stresses and strains is well-known as the shear modulus \(\mu\):

\[
\tau_{ij} = 2\mu \epsilon_{ij} = 2\mu \epsilon_{ij},
\]

which gives

\[
\mu = \rho b^2 \delta_{\delta_j}.
\]

This definition agrees with that of the shear sound velocity as

\[
\mu = \rho b^2.
\]

The relaxation time function is chosen in the following form [9]:

\[
\tau(\rho, s, T) = \tau_0 \exp \left[ \frac{(D_0 + HE_s)}{N_0 + M\epsilon} \right],
\]

where, \(s^2 = s_{ij} s_{ij}\). Here, \(\tau_0\) and \(D_0\) (a drag stress) can be functions of temperature [23] when the rate sensitive yield limit data at elevated temperatures are available. When considering hardening, \(\epsilon\) should be a measure of the plastic or total deformation [24] in accordance with the Orowan relation of the dislocation dynamics. However, the involvement of the plastic deformation as an independent variable poses significant thermodynamic difficulties against the model. Recently, Prof. Romensky has suggested another measure of plastic deformation in (19) that enables one to successfully model the hardening. This measure is associated with a function of entropy that is a characteristic of plastic work [25]. In [9] and in the present work the variable \(\epsilon\) is associated with the elastic portion of the deformation \(\epsilon = s/(2\mu)\) and \(\tau_0, D_0, H, N_0, M\) are material constants.

### 4. Thermodynamic Correctness

In order to verify the thermodynamic correctness we consider the specific internal energy (15) in the following general form:
Expanding the energy conservation law in (9), the momentum conservation law will annihilate the kinematic term of the full energy in the expansion and we have the following energy balance equation:

$$\frac{dE}{dt} - s_{ij} \frac{\partial u_j}{\partial x_i} + p \frac{\partial u_k}{\partial x_k} = 0 .$$  \hspace{1cm} (20)

Taking the substantial derivative of (20) and using the chain rule, we can compare the result of differentiation with (21), which gives

$$\rho \frac{dE}{dt} = \rho E \frac{d\rho}{dt} + \rho E_{e_i} \frac{de_i}{dt} + \rho E_s \frac{dS}{dt} = s_{ij} \frac{\partial u_j}{\partial x_i} - p \frac{\partial u_k}{\partial x_k} .$$

Using (11) and replacing the substantial derivatives for density and elastic deformations from (9), we have

$$\rho T \frac{dS}{dt} = s_{ij} \frac{e_{ij}}{\tau} .$$  \hspace{1cm} (22)

Thus, the entropy production $dS/dt = \Phi$ is non-negative as soon as $s_{ij} e_{ij} \geq 0$, because $\Phi = (s_{ij} e_{ij})/(\rho T \tau)$. The shear modulus in (17) may depend on density or some internal variables, but it is non-negative. Therefore, $\Phi = (s_{ij} e_{ij})/(\rho T \tau) = 2 \mu (e_{ij} e_{ij})/(T \tau) \geq 0$.

Finalising the correctness analysis, we will check out the hyperbolicity of the system restricting the consideration to the one-dimensional case for the sake of simplicity. The system of equations (9) in the one-dimensional case can be written down as follows:

$$\frac{dp}{dt} + \rho \frac{\partial u}{\partial x} = 0 , \quad \frac{de_{11}}{dt} - \frac{2}{3 \rho} \frac{\partial u}{\partial x} = - \frac{e_{11}}{\tau} , \quad \frac{de_{22}}{dt} + \frac{1}{3 \rho} \frac{\partial u}{\partial x} = - \frac{e_{22}}{\tau} ,$$

$$\frac{du}{dt} - \frac{1}{\rho} \frac{\partial s_{11}}{\partial x} + \frac{1}{\rho} \frac{\partial \rho}{\partial x} = 0 , \quad \frac{dS}{dt} = \Phi ,$$

here $u = u_1$ and $x = x_1$. In the matrix form this system is

$$\frac{dV}{dt} + A \frac{\partial V}{\partial x} = B ,$$

where
The characteristic equation

$$|A - \lambda I| = 0$$

can be rewritten as

$$\chi^2 \left[ \chi^2 - \left( p + \frac{2}{3\rho^2} (s_{11})_{e_1} - \frac{2}{3\rho^2} (2p_{e_1} - p_{e_2}) \right) \right] = 0. \quad (24)$$

Here $\chi = u - \lambda$. It can be easy seen from (23) that three zero roots of the characteristic equation (24) should identify contact characteristics, along which the combinations $S, e_1 + 2e_2$, and $1/\rho - e_1 + e_2$ are preserved. The second contact condition is actually an identity from (7) and the last condition actually involves highly variable right-hand sides. Specifically, from (23) this condition can be formulated as follows:

$$\frac{d}{dt} \left( \frac{1}{\rho} - e_1 + e_2 \right) = \frac{e_1 - e_2}{\tau}. \quad (25)$$

This equation is an analogue of equation for the combination $\ln(\rho) + e_1 - e_2$ from the prototype model (1), (5). The equation (25) actually describes the wave structure within the internal viscosity zone because the corresponding contact condition does not link states before and after a real jump but it connects states within some plasticity transition zone (see [5]). This third condition for $1/\rho - e_1 + e_2$ could be replaced by an equivalent condition for $1/\rho - 3e_1/2$. Corresponding constitutive equation for $e_{22}$ in (23) does not need to be calculated due to $e_1 + 2e_2 = 0$.

Thus, from (24) it follows that the sound characteristics exist when

$$p + \frac{2}{3\rho^2} (s_{11})_{e_1} - \frac{2}{3\rho^2} (2p_{e_1} - p_{e_2}) \geq 0. \quad (26)$$

The condition (26) is actually the hyperbolicity condition. The derivative $a^2 = p_{\rho}$ is known as the bulk sound velocity and from (17): $(s_{11})_{e_1} = 2\mu$, where the shear modulus $\mu$ is associated with the shear sound velocity via (18). The derivatives in the last three terms of (26) are of the second order of magnitude because from (15-17):
\[(s_{11})_\rho = (2\mu \rho)^{-1} e_{11}, \quad p_{\epsilon_{11}} = p_{d_{11}} = p_{e_{11}}, \quad p_{\epsilon_{22}} = p_{d_{22}} = p_{e_{22}}.\]

When ignoring those terms, the condition (26) takes the following form from (17-18)

\[c^2 = a^2 + \frac{4b^2}{3} \geq 0,\]

that agrees with the definition of the longitudinal sound velocity for \(c^2\), similarly to (14).

Thus, the hyperbolicity condition agrees with the well-known restrictions \([4, 5]\) that are applicable to the present formulation of the model, too.

### 5. CTH Implementation

The present implementation affects the three CTH blocks mentioned earlier: IB, LB, and ERB modules.

#### 5.1 Input Block

For the first IB part of the code three subroutines \([11, 12, 26]\) were involved in the modifications, namely UINEP.FOR, UINISV.FOR, and UINCKH.FOR.

**5.1.1 UINEP modifications**

UINEP.FOR modifications read in the data from VP_data input file into the VPUINP array allocated for the EP related input data \([26]\). The data needed for the model input are: ‘RHO’, ‘C0’, ‘B0’, ‘ALF0’, ‘BET0’, ‘GAM0’, ‘CV’, ‘TAY0’, ‘D0’, ‘H0’, ‘AN0’, and ‘AM0’.

These constants represent the following:
- initial density for RHO (\(\rho_0\) in (15));
- the bulk sound velocity for C0 (\(a_0\) in (15));
- the shear sound velocity for B0 (\(b_0\) in (15));
- the bulk modulus exponent for ALF0 (\(\alpha_0\) in (15));
- the shear modulus exponent for BET0 (\(\beta_0\) in (15));
- the Gruneisen coefficient for GAM0 (\(\gamma_{0}\) in (15));
- the thermal capacity for CV (\(c_v\) in (15));
- the relaxation time coefficient for TAY0 (\(\tau_{0}\) in (19));
- the drag stress for D0 (\(D_0\) in (19));
- the hardening parameter for H0 (\(H\) in (19));
- the initial dislocation density for AN0 (\(N_0\) in(19)); and
- the dislocation multiplication coefficient for AM0 (\(M\) in(19)).

It should be noted that the model uses non-standard input units in cm (length), g (mass), 10 \(\mu\)sec = 10\(^{-5}\)sec (time), and °K (temperature). The derived pressure unit in this case is GPa. At the end of the modifications, the initial values for Poisson ratio, and the bulk and shear modulus are calculated and initially checked in the same UINEP.FOR subroutine.
5.1.2 UINCHK modifications

UINCHK.FOR modifications call new subroutines VEMCHK.FOR and VEMCHX.FOR. Before the calls, the modification code checks if the assigned MODLEP number for the present constitutive model is in agreement with the substituted EOS number MEQ [26]. Next part of the modifications arranges definition of necessary types of the model allowing one to treat the model as the one, for which deviatoric stresses will be calculated. Then, a standard call to the subroutine SI2CTH introduces the unit transformation constants into a part of the input array VPUINP. The new subroutine VEMCHK.FOR transforms the constants into the CTH units from the non-standard input units, introduces global constants (the ‘GC’ part [26] of the array VPUINP) such as initial temperature used in (13) and numerical limiting constants for calculation of constitutive equations, and fills in the ‘DC’ part of the array with several derived constants used for EOS and CE calculations. The new subroutine VEMCHX.FOR called afterwards replaces the EOS input data (UI) by the data from the VP_data file, and recalculates auxiliary constants for the following EOS calculations filling tail of the UI array for the EOS block [26] by these constants.

In order to allow for the decoupling, subroutines available for an EOS one-component model are replaced by the subroutines representing the present EOS (13) or (15). Therefore, an identifier previously used for the model from the CTH EOS-database in an EOS section of the CTH input is now used for the present model. Thus, the identifier and any dummy material name from available material database for that CTH’s EOS model identify the EOS model described in a previous section. The EOS constants previously taken from the EOS analogue of the VP_data file are now replaced by the data described above in the present section, which is done when calling VEMCHX.FOR. However, the code uses the EOS data for initial calculation of mass arrays because input of EOS data directly followed by the mass calculation is processed earlier than the elasto-plastic input. In order to use the input data correctly, when using the EOS identifier in the EOS section of the CTH input, we assign explicitly value of the initial density for the EOS model to be substituted, while leaving the remaining parameters of the EOS input intact. This value should be in agreement with the value of ‘RHO’ in the VP_data input.

5.1.3 UINISV modifications

UINISV.FOR modifications make two calls for a standard subroutine MIGSEX [26] setting up default values for extra variables and for a new subroutine VEMEXV. The subroutine VEMEXV sets up one extra variable for strain deviator $D$ in (13) and declares it to be scalar and non-convecting with selection of proper values for ITYPE and IADVCT (see [26]). The variable is dimensionless that is declared by zeroing the array RDIM [26].

5.2 Lagrangian Block

For the second LB part of the code, one subroutine ELSG [11, 12, 26] is involved in the modifications.
5.2.1 ELSG modifications

Modifications to the ELSG subroutine is the main CE driving part of the model. This part processes the equations taken in the standard form [27] for the Jaumann derivative describing evolution of the stress tensor. The deviatoric part of the stress tensor is an analogue of the equation (6) of the model. The first part of the modifications in ELSG extracts the stress deviators at an old time step and the strain rate increments at the advanced half time step [27]. Next, new subroutine VEMDRV is called that calculates new stress deviators. To do so, the subroutine first calculates the stress deviators in accordance with the strain deviator similar to the constitutive equations (5) or (6) when ignoring the relaxation right-hand terms. Thus, an intermediate stress deviator \( s^\#ij \) is calculated. Next step is dealing with the stress reduction due to relaxation. For calculation of the scaling factor of the stress deviator, the equation (6) is reduced to the following one for the parameter \( s \) from \( s^2 = s_{ij} s^{\#ij} \):

\[
\frac{ds}{dt} = -\frac{s}{r} \quad (27)
\]

Here, the function \( r(s) \) is determined in (19). The equation (27) is discretised as follows

\[
\frac{s^{n+1} - s^\#}{\Delta t} = -\frac{s^{n+1}}{r}(s^{n+1}) \quad (27)
\]

where \( \Delta t \) is the time step and \( s^\# \) is calculated from the intermediate stress deviator \( s^\#ij \). New value of the parameter \( s^{n+1} \) is calculated from (28) by iterations, using Newton’s method. The stress deviators are calculated afterwards in the standard fashion:

\[
s_{ij}^{n+1} = \left( s_{ij}^{n+1} / s^\# \right) \cdot s_{ij}^{n} \quad .
\]

Potentially, a value of the extra variable could be calculated at this stage and then, subject to a weighted convection, transferred to the ERB module of the code using the extra variable array allocated in the UINISV.FOR subroutine. However, we fill this array with the \( D \)-values only for debugging control, passing it on to the Eulerian remap block.

5.3 Eulerian Remap Block

The third and last modification in the ERB block deals with the EREB subroutine [11].

5.3.1 EREB modifications

These modifications deal with EOS calculations during the Eulerian remap step. In order to properly use the extra variable \( D \), a modification to the EREB subroutine includes calculation of \( D^{n+1} \) from the remapped values of \( s_{ij}^{n+1} \) and density \( \rho^{n+1} \) entering the shear modulus \( \mu \) according to the following formula used for (13):

\[
D^{n+1} = s_{ij}^{n+1} \cdot s_{ij}^{n+1} / [2 \cdot (2\mu)^2] \quad .
\]
where $\mu$ is taken from (16, 18). The variable $D$ can easily be accessed as an extra variable for the CE block and stored in the corresponding array. However, the EOS block employs its own extra variable array. Therefore, next step of the modifications is writing the CE extra variable array to the EOS extra variable array. Now, the extra variable can be used in the EOS subroutine that is called from the EREB subroutine. Access to the EOS subroutine is available only through the substitution subroutines (parts of the EOS subroutine) that calculate necessary pressure, temperature and energy parameters along with the necessary derivatives from (13) or (15). Pressure and temperature are calculated from (11). These modifications to the ERB module finalise implementation of the model.

### 6. Modelling of Shock Behaviour

One-dimensional calculation of the shock-wave propagation is the basic test to check out how accurately the strength effects are modelled.

![Figure 1](image_url)  
**Figure 1**  
Shock wave propagation in Iron (a), Aluminium (b), and Copper (c) (0.5 $\mu$sec output frequency)

CTH calculations of the velocity profiles resulting from the collision of two metallic plates (1 cm-thickness flyer plate against 2 cm-thickness target made from the same material) are shown in Fig. 1 for Armco-Iron, Aluminium (a low-impurity alloy), and Copper. The material constants for EOS (13) and the constitutive function (19) are taken from [9, 21, 28].

The computation results demonstrate that the numerical viscosity of the CTH scheme does not destroy the wave structure solution. The start and evolution of the elastic precursor are seen quite clearly. Comparison of this solution with available calculation [6], reproduced in Fig. 2 for the shock propagation in Iron, shows a good agreement.
Thus, the fine wave structure along with the strength feature, manifesting itself as the residual velocity of the flyer plate after reflection of the rarefaction wave from the free surface, can be described quite well with the CTH hydrocode. It should be noted that this is not a trivial problem with commercial hydrocodes. For example, many attempts conducted in DSTO to model the separation of the elastic precursor from the plastic wave using a 2D-version of the DYNA hydrocode have failed because of a high dumping effect of the code’s numerical viscosities.

7. Ballistic Impact and Taylor Test Modelling

Ballistic impact is a basic problem for evaluation of the weapon effectiveness. The numerical results below describe a 1 km/s-velocity impact of the cylindrical steel projectile (1 cm-diameter and 1 cm-height) against 5 mm-thickness Aluminium plate target.

For this case, the plug formation and its separation from the target is a typical feature of the process. To simulate the plug separation correctly, fracture modelling is normally needed. Fracture modelling might be arranged for the present model within the CTH implementation framework (the fracture implementation in CTH is usually of the viscous type, in contrast to the brittle type implementations used in the code [7] and in the DYNA-implementation [9]). However, the fracture modelling will be neglected in the present report. It should be noted that the material erosion observed frequently in Eulerian calculations, including the present ones, is a numerical artefact and it is not associated with a physical fracture process.

The model [4, 5] has been employed in the calculations [7, 24] shown in Fig. 3 obtained with a code based on the Lagrangian-Eulerian technique [29] and in the calculations [9] shown in Fig. 4 obtained with the Lagrangian version of DYNA.
Both calculations employed the split-element algorithm [9] that introduces explicit Lagrangian free surfaces inside the material; this splitting is subject to fulfilment of a fracture criterion. Due to early release of the plug from the target and arrival of the release waves into the rear side of the projectile the both calculations show some bulging of the projectile backwards at the symmetry axis, which is observed with larger extent for the results in Fig. 3. A fairly good preservation of the projectile shape is also observed, where the better preservation for the results in Fig. 4 is explained by a finer grid and, correspondingly, less numerical viscosity.
Figure 5  CTH calculation of the ballistic impact

The present calculation shown in Fig. 5 is characterised by a longer unreleased interaction between the projectile and the target when compared with the calculations in Figs. 3, 4 that take into account the crack propagation effects. Therefore, the projectile is slightly spread out along the target surface and does not show signs of the bulging backwards at the rear free side of the projectile in the proximity of the centreline. In contrast, a slight surface dip is observed.

In order to trace convergence of the solution, another calculation has been conducted with a mesh that is 5-times finer in each direction. The results are shown in Fig. 6 and demonstrate quite close results to those in Fig. 5.
Less effect of the numerical multi-material diffusion is noticeable in Fig. 6 with slightly less spread out of the projectile and erosion of the cavern’s edges, when comparing with the results of Fig. 5. The both calculations show the slight dip at the projectile’s free surface.

In order to find if the features of the solution are related to the model, we have conducted the Taylor test calculations. Taylor test is used for the model validation because it enables one to compare the residual shape of a projectile collided against anvil with the shape calculated with a material model. Results of the Taylor test calculation in Fig. 7 are for a copper projectile impacted with a velocity of 200 m/s. The results along with the von Mises model calculation at $Y = 0.2$ GPa are zoomed in for the last frame in Fig. 8.
The von Mises model calculation of Fig. 7 has been conducted with the yield limit $Y$ to be 0.1 GPa that is a value corresponding to a low strain rate (approximately $10^{-2}$ 1/sec) for the strain rate range, within which the present model operates. The comparison of the results in Fig. 8 along with the high strain rate ($10^3$ 1/sec) yield limit value of 0.2 GPa (Fig. 8(b)) shows that the change of the yield limit is an important factor and this variation may result in both bulging and spreading out of the projectile. In general, the calculation of Fig. 8(b) approximates the calculation with the present model better, distorting, nevertheless, the projectile form-factor and surface dipping slightly more than the present model calculation. Therefore, it is reasonable to expect that the dip effect in the calculations of Figs. 5 and 6 is associated with the fracture neglect rather than with the model.

8. Shaped Charge Jet Modelling

The Shaped Charge (SC) is another important element of warheads and specialised weaponry, which needs to be assessed. In the present section, we consider formation of SC jet from 3 mm-thickness 90º-Aluminium liner which is, along with Aluminium 3 mm-thickness shell, encasing a High Explosive (HE) Charge of Composition B. The SC parameters are 76 mm in diameter, and 36 mm from the liner apex to the detonator located at the centre-point of the rear side of the charge.

Initial stage of the calculation at 5 $\mu$sec after initiation in Fig. 9 shows propagation of the detonation wave. The remaining graphs show development of the SC jet with the interval spacing one frame from the next one of 5 $\mu$sec in Fig. 9 and of 4 $\mu$sec in Fig. 10.

The jet formation in the present case is an intermediate regime between the inverse jet formation (e.g., for liners with the cone angle nearly 120º [8]) and the cumulative jet formation typical for the small cone angles. This regime is interesting because it provides both significant mass of jet and large jet’s kinetic energy.
Figure 9  Shaped Charge. Initial stage of the jet formation.

The SC jet formation has been calculated on a uniform mesh and on a mesh controlled by the Advanced Mesh Refinement (AMR) option [30].

Figure 10  Shaped Charge. Final stage of the jet formation.

The both results are very close to each other with the exception of the tail feature at 44 μsec in Fig. 10 of the slug formation. This feature is frequently attributed to an effect of material viscosity in numerous SC calculations with visco-elastic models. This effect was observed in the uniform mesh calculation but it is not observed in the AMR calculation.
9. Hyper-Velocity Impact Modelling

The last set of calculations for validation of the model implementation simulates the hyper-velocity impact modelled earlier in [7] with the use of model [4, 5]. Impact by steel projectiles was considered in [7] at various angles of impact $\alpha$ from $90^\circ$ (normal impact) to $30^\circ$. The objective was to analyse the influence of the angle of impact on the behind target debris formation. In particular, the impact at small angles of impact was shown to be more dangerous to the behind target witness plate than the normal impact. The major reason was shown [7] to be fragmentation of the projectile into smaller particles at the normal impact and into fewer and much larger particles at the oblique impact.

The publication [7] has employed the accumulation damage criterion for the fracture analysis of the problem. It has been observed in [7] that the projectile at the oblique impact is in contact with the target for a longer time, which prevents the projectile from the stress release and accumulation of damage. At the same time, the normal impact results in a quick release, that results in a fast accumulation of damage and dispersing the projectile in small fragments. The present consideration ignores the fracture analysis. However, we can observe the character of the projectile-target interaction and judge if the contact is long enough to prevent the projectile from the stress release.

The CTH calculation results are compared with DYNA calculation and experiment [19] in Fig. 11. X-ray photography of the SC jet has been taken at the time, when the jet has travelled approximately 100 mm below the Shaped Charge base. The comparison in Fig. 11 demonstrates an excellent agreement of the present calculation (in contrast to the DYNA calculation) of the jet shape and SC jet parameters with the experiment.
As in [7], we consider the impact of 2.3 mm-diameter steel spherical projectile against 0.5 mm-thickness Aluminium target with the impact velocity of 5 km/s.

Figure 12 Calculation of hypervelocity Impact [7] (α = 90°). 2D-plane set-up

Calculation of the normal impact in the plane 2D-set-up is shown in Fig. 12 (reproduced from [7]) and in Fig. 13 (the present CTH calculation).

Figure 13 The present CTH calculation of hypervelocity impact (α = 90°). 2D-plane set-up.
Figs. 14 and 15 show 2D-axisymmetrical calculations within the same set-up.

Figure 14 Calculation [7] \((\alpha = 90^\circ)\) in the 2D-axisymmetrical set-up (central cross-sectional view)

In fact, because of the plane set-up, the calculations in Figs. 12-13 simulate the lateral rod impact rather than the spherical particle impact.

Figure 15 CTH 2D-axisymmetrical calculation \((\alpha = 90^\circ)\) (central cross-sectional view)

Comparison between the results of the plane and axisymmetrical calculations (Figs. 12-13 and Figs. 14-15, respectively) shows that for the case of axisymmetrical set-up, a more compact projectile after the penetration and thinner plug due to the smaller target mass involved in the
interaction with the projectile are observed both in the calculation [7] shown in Fig. 14 and in the present CTH calculation (Fig. 15).

It should be noted that the visible fragmentation in Figs. 13 and 15 is not associated with the material fracture. The multi-material diffusion effect called numerical erosion when the target material travels through the behind-target space causes this pseudo-fracture. However, the CTH calculations confirm that the release wave propagates back to the projectile very quickly after the start of the projectile-target interaction (before 0.5 μsec). In fact, it occurs even before the projectile separates the plug from the main body of the target due to a small mass of the target interacting with the projectile in the normal direction.

![Figure 16](image)

Figure 16  The present CTH calculation of hypervelocity impact ($\alpha = 90^\circ$). 3D-set-up (central cross-sectional view)

Oblique impact cannot be calculated in the axisymmetrical set-up. Therefore, for the sake of consistency the CTH calculation of Fig. 15 has been conducted in the three-dimensional set-up as well. Results of the 3D-calculation cross-sectioned for a better visibility are shown in Fig. 16.
The present CTH calculation of hypervelocity impact ($\alpha = 90^\circ$). 3D-set-up. Angle views at the final stage

The full-scale view of the same calculation is shown in Fig. 17. The left graph shows the view from the target side, where the projectile was launched from, and the right graph shows the view from the behind-target area.

Comparison of the 2D-calculation in Fig. 15 with the 3D-calculation in Fig. 16 demonstrates a fair correlation between the results. It should be noted that the 3D-calculation has been conducted on the same mesh in X- and Z-directions as the 2D-calculation. Therefore, the differences observed are associated with a larger mass of numerical cells for the axisymmetrical case than for the 3D-case. In that sense, the axisymmetrical set-up is dealing with a rougher mesh than the three-dimensional set-up. This is resulting in a slightly larger mass diffusion for the axisymmetrical set-up in Fig. 15 and, as a result, in a thinner plug, when comparing with the results in Fig. 16.

Numerical calculations at the moderate angle of impact ($\alpha = 60^\circ$) has not been conducted in [7] because the experimental results demonstrated that the projectile fragmentation is significant for this case and no impact abnormality has been observed. However, we can conduct this calculation with CTH in order to confirm that the projectile-target interaction does not manifest any features resulting in reduced accumulation of damage.

The CTH calculation results for the case of $\alpha = 60^\circ$ in the three-dimensional set-up are shown in Fig. 18 as the cross-sectional view, where the cross-section passes through the symmetry plane of the set-up. Fig. 19 shows the full-scale view and the graphs in the figure have the same arrangement as in Fig. 17.
Figure 18  CTH calculation of hypervelocity impact (α = 60°). 3D- set-up (central cross-sectional view)

Figure 19  The present CTH calculation of hypervelocity impact (α = 60°). 3D-set-up. Angle views at the final stage
For the oblique impact ($\alpha = 30^\circ$), the 2D-code [7] can deal only with the plane set-up. The corresponding calculation results are shown in Fig. 20.

\[ t = 0.84\mu s \quad t = 1.14\mu s \]

**Figure 20** Calculation of hypervelocity Impact [7] ($\alpha = 30^\circ$). 2D-plane set-up

The CTH calculation results within the same set-up are shown in Fig. 21.

**Figure 21** CTH calculation of hypervelocity impact ($\alpha = 30^\circ$). 2D-plane set-up

The CTH calculation results within the same set-up are shown in Fig. 21.
A full-scale 3D-calculation conducted with CTH gives the results shown in Figs. 22 and 23.

The calculations in Figs. 21-23 have been conducted on a uniform mesh. In order to check out the mesh effects the AMR [30] calculation has also been conducted and summarised in Figs. 24 and 25.
Figure 24  3D-CTH AMR calculation of hypervelocity impact ($\alpha = 30^\circ$) (cross-sectional view)

Figure 25  3D-CTH AMR calculation ($\alpha = 30^\circ$). Angle views at the final stage
The results in Figs. 18-19 for $\alpha = 60^\circ$ show that duration of the projectile-target interaction is actually of the same order as for the normal impact with a certain stress release from the interaction occurring by 0.75 $\mu$sec. At the same time, the case $\alpha = 30^\circ$ demonstrates that the projectile-target interaction lasts up to 1.5 $\mu$sec after the start of the interaction. Even the numerical erosion consideration in the figures above shows that much larger area of the target follows the projectile and the projectile leaves the target area completely only after 1.5 $\mu$sec.

10. Conclusions

The present work reports on a procedure that allows us to formulate a material model decoupling the shear and volumetric responses of material. It is believed that this procedure could be extended to other models dealing with strength effects. This formulation is suitable for implementation in the CTH hydrocode.

A Maxwell-type viscoelastic model that describes behaviour of conventional materials manifesting rate sensitivity is reformulated with the use of the proposed decoupling procedure. Thermodynamic correctness of the model has been verified.

The model has been implemented in CTH and the implementation steps described briefly according to the code flowchart available in the literature.

The model implementation has been validated against available calculations within the prototype model using a specialised 2D-code developed earlier and/or a DYNA-implementation of the prototype model. The CTH results correlate well with the calculations and in a number of cases the present results describe experiments with a greater detail than the calculations available in the literature.

Results of the present report have demonstrated that the basic CTH implementation capability has been established in DSTO. However, the future models relying on a wider involvement of the extra variables in the model implementation require a more intrusive implementation in the ERB part of the code, which may require more advanced implementation strategies and a modification of additional components of the code.

Acknowledgements

The author is grateful to Dr. Scott Schoenfeld of ARL and Dr. Gene Hertel of SNL for support in the release of the CTH hydrocode to DSTO, to Drs. David Crawford and Bob Schmitt for setting up a comfortable CTH environment in DSTO and useful communications, and to Dr. Shane Schumacher for useful communication on the implementation issues in the Eulerian block of the code.
References


25. Romensky E.I., Private communication, 2007, Cranfield University, UK.


Thermodynamically Consistent Decoupled Shear-Volumetric Strain Model and CTH Implementation

A.D. Resnyansky

DSTO Defence Science and Technology Organisation
PO Box 1500
Edinburgh South Australia 5111 Australia

Technical Report
June 2009


Chief, Weapons Systems Division

APPROVED FOR PUBLIC RELEASE

Many hydrocodes, such as LS-DYNA and CTH, require the decoupling of the shear from volumetric response in a material model used. A constitutive model is formulated, which decouples the responses of a rate sensitive material. Basis of the model is a general Maxwell-type viscoelastic model, which, however, is not originally decoupled and, thus, not suitable for implementation in the hydrocodes. The formulation provides the thermodynamic consistency for the case of small deviatoric elastic deformations and unrestricted volumetric response. A schematic of implementation in CTH is briefly described. Numerical illustrations demonstrate agreement of the CTH calculations with calculations available in the literature.