

# Feasible GTEM baselines of the world economy for the next 100 years

*Hom Pant, Brian S. Fisher and Vivek Tulpulé*  
Australian Bureau of Agricultural and Resource Economics

---

6<sup>th</sup> Annual Conference on Global Economic Analysis  
The Hague, June 12-14, 2003`1

What are the possible ‘business-as-usual’ growth rates of the world economy for the next 100 years and what are the likely patterns of regional growth? The answers to these questions are important for many reasons, principal among them is that the world and regional economic growth rates are primary factors determining global emissions growth rates that are expected to drive future climate change. In addition, the growth in the size of future markets for commodities depends much on the growth rates of regional economies. Hence, a knowledge of possible future growth rates is important to both investors and producers. This paper addresses these questions using the intertemporal version of ABARE’s Global Trade and Environment Model (GTEM). In particular the paper concludes that although there seems to be no restriction on the growth rate along the balanced growth path, there are limits to economic growth that are driven by region and factor biased technical progress and factor accumulation.

ABARE project 2782  
ISSN 1447 3666



abareconomics

## 1 Introduction

It is commonly understood that economic growth not explained by factor accumulation must be explained by productivity growth, which is either induced by policy change or by technological change or both. Both changes in policies and productivity, however, are normally treated in general equilibrium models, such as GTEM, as exogenous. Hence, given the initial state of the world economy and *assumptions* regarding the nature and extent of policy changes and technological progress over time, one can, in principle, fully determine the path of the world and regional economic growth based on the projections from dynamic general equilibrium models. In practice, questions regarding the paths of future economic growth boil down to specifying the nature and extent of policy (institutional) changes and specifying the pattern and extent of technological change.

Normally, policy changes and institutional reforms are the means by which policy makers hope to influence and guide the future course of economic development. Alternative policy changes and institutional reforms are ranked by their deviations against the baseline of the economic system. Therefore, generation of a reasonable baseline becomes the primary task in any serious study of factors influencing economic dynamics and growth.

The concept of the baseline itself needs an operational definition. We define the baseline as a growth trajectory in which no unannounced policy changes take place. In other words, we do not let policy variables change in the baseline unless they have already been announced in the past or in the historical years – such as parameters set out by trade agreements aiming to reduce specific trade barriers to pre-specified levels by certain dates in the future. Even in the absence of unannounced policy changes, the world does not remain stationary. It will change because of the momentum of past shocks. It may also change because of induced technological progress, as economic incentives change over time as population changes, as resources are depleted and as relative prices change. The world may also change because of some autonomous or random change in production technologies triggered by new discoveries independent of economic incentives or constraints. So our operational definition of the baseline includes the momentum of the system, announced policy changes and induced as well as autonomous technological changes that may take place over the horizon of the simulation.

A key issue relating to autonomous technical change is that, in principle, it is possible to imagine an infinite combination of autonomous productivity changes taking place over time in different regions. The question that then arises is whether the regions can have any type and size of productivity growth or is there a limit to the nature and extent of the change? Here we encounter another problem in choosing a criterion for deciding whether a particular trajectory of productivity growth or factor accumulation is feasible or not. We declare a trajectory of exogenous variables *infeasible* if it does not lead to the *steady state* of the global economy. Hence whether the model converges to a solution or not is the criterion used to decide upon the limits to growth along the baseline of the world economy. This paper seeks to answer the question whether there is any limit to economic growth of the global economy?

This paper is divided into 8 sections. In section 2, we provide the characteristics of GTEM that identify it as a neo-classical growth model of the world economy. In section 3 we review the standard Solow-Swan model and in section 4 we review an open economy version of the Solow-Swan model to see whether it implies any restriction on economic growth. In section 5 we draw implications from the Solow-Swan model for multi-sector, multi-region models of the global economy, such as GTEM. In section 6 we try to answer the question whether there is any speed limit for the global economy. In section 7 we explore the existence of speed limits through simulations of GTEM and conclusions are drawn in section 8.

## 2 Intertemporal GTEM as a neoclassical growth model of the world economy

Intertemporal GTEM is a forward-looking dynamic multi-sectoral and multi-regional model of the global economy. Production sectors combine land, labor, natural resources and physical capital using a CES function into a factor composite. They also combine fuel commodities using a CES function into a fuel composite. The fuel and factor composites are again combined using a CES function to form a fuel-factor composite, which is finally combined with non-fuel intermediate inputs in fixed proportion to produce output. This nested structure of production functions provides some degree of flexibility in obtaining optimal input combinations as relative input prices change.

Supply of land and natural resources are exogenously determined and the capital accumulation process endogenously determines the supply of capital services. As the labor saving technological process is exogenously determined, the effective supply of labor is also exogenously given and for this exercise the total population of each region and its growth rate is also taken as exogenously given although, in standard GTEM, changes in population are determined endogenously. Hence in this version of the model, the supplies of labor, land and natural resources are exogenously given and the supply of capital services is endogenous.

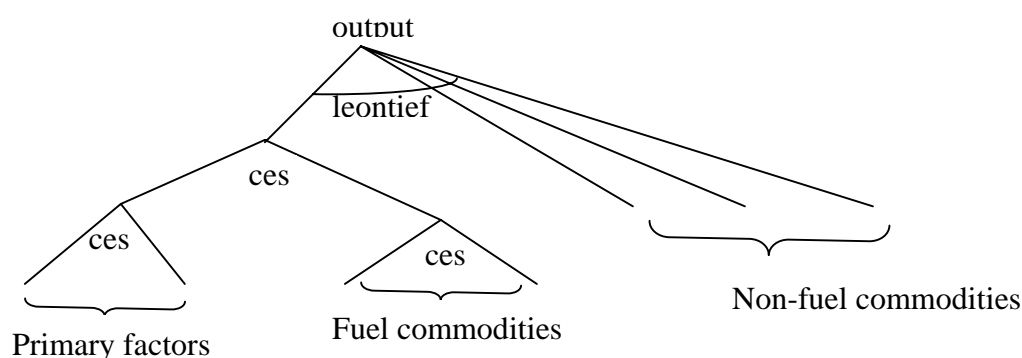


Figure 1: production structure in GTEM

Infinitely lived households receive all factor incomes and tax revenues. They are assumed to maximise the discounted sum of utility subject to their intertemporal budget constraint. The assumption that households have static expectations of prices and of other relevant variables imply that their saving rates are fixed.

As the world economy is closed by definition, and given that constant returns to scale holds for all production functions in all sectors in all regions together with the assumption of fixed savings rates, GTEM satisfies the conditions of the Solow-Swan model. It is however important to note that the standard Solow-Swan model has one sector while GTEM is multi-sectoral. The Solow-Swan model has two factor of production, while GTEM has four of which the supply of land, labor and natural resources are exogenous. Hence, it is quite instructive to review the Solow-Swan model to obtain insights into the steady state properties of GTEM. While reviewing the Solow-Swan model we will ignore the fact the GTEM has four factors of production and it uses intermediate inputs as well.

### **3 The Solow-Swan growth model of a closed economy**

Let the production function of a one-sector closed economy be described by a CES function as:

$$(1) \quad Y = G(L, K) = [aL^{-\phi} + (1-a)K^{-\phi}]^{-1/\phi}; \quad \phi > -1 \text{ and } \phi \neq 0.$$

where L is the effective unit of labor, K is the effective unit of capital services and  $0 < a < 1$ .  $\sigma = 1/(1+\phi)$  gives the elasticity of factor substitution between the two factors, L and K. Assume that labor supply L follows the exogenous growth path:

$$(2) \quad L_t = L_0 e^{nt},$$

where n is the constant (annual) growth rate of the effective labor supply, and  $L_0$  is normalised to unity. By definition n captures both physical and productivity growth of the effective labor force. Writing the production function in per capita (per effective labor units) terms, and denoting the per capita variables by lower case letters, we get

$$(3) \quad y = g(k) = [a + (1-a)k^{-\phi}]^{-1/\phi}.$$

As the economy is closed, the saving-investment equality holds at all times and the households are assumed to save a fixed fraction,  $s$ , of their income, which means we have

$$(4) \quad dK / dt + \delta K = sY,$$

where  $\delta$  is the depreciation rate of physical capital. Writing (4) in per capita terms, we obtain

$$(5) \quad \frac{dk}{dt} + (n + \delta)k = sg(k)$$

Steady state (SS) is defined as a situation whereby we obtain

$$(6) \quad \frac{dk}{dt} = 0,$$

which means at steady state we must have

$$(7) \quad (n + \delta)k^* = sg(k^*).$$

To help describe the steady state equilibrium graphically, following Barro and Sala-i-Martin (1995), equation (5) is rewritten as

$$(8) \quad \gamma_k = \frac{dk/dt}{k} = \frac{sg(k)}{k} - (n + \delta)$$

where  $\gamma_k$  is the growth rate of the per capita capital stock. As can be seen by differentiation  $d(g(k)/k)/dk < 0$  and the right hand side of equation (8) can be graphed as in figure 2.

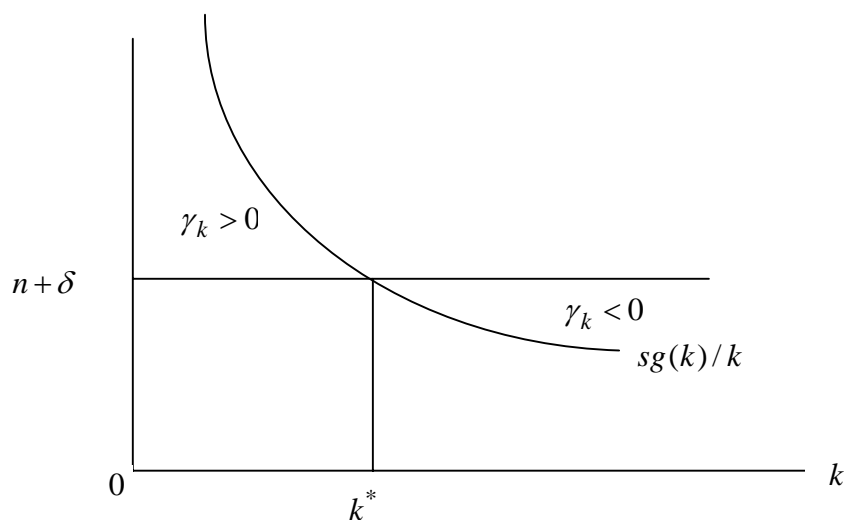


Figure 2: Steady state in Solow-Swan model

Figure 2 illustrates the intersection between the curves labelled  $sg(k)/k$  and  $n + \delta$  at an interior point giving the steady state value of per capita capital services at  $k^*$ . At this point the growth rate of  $k$  is zero. To the left of this,  $\gamma_k > 0$  and to the right of  $k^*$  its growth rate is negative. Hence the equilibrium is stable.

It is well known that in order to guarantee an interior solution the production function has to satisfy the following conditions – known as Inada conditions:

$$(i) \quad g(0) = 0; \text{ and } \lim_{k \rightarrow 0} g'(k) = \infty$$

and

$$(ii) \quad g(\infty) = \infty; \text{ and } \lim_{k \rightarrow \infty} g'(k) = 0.$$

Violation of the first condition may lead to a situation with  $k^* = 0$  and violation of the second condition may lead to endogenous growth in the economy.

It follows from equation (3) that for a CES function

$$(9) \quad g(k)/k = [ak^\phi + (1-a)]^{-1/\phi}.$$

Hence, we have:

$$(10a) \quad \lim_{k \rightarrow 0} g(k)/k = \lim_{k \rightarrow 0} g'(k) = \begin{cases} (1-a)^{-1/\phi} & \text{if } \phi > 0 \\ \infty & \text{if } \phi < 0 \end{cases}$$

and,

$$(10b) \quad \lim_{k \rightarrow \infty} g(k)/k = \lim_{k \rightarrow \infty} g'(k) = \begin{cases} 0 & \text{if } \phi > 0 \\ (1-a)^{-1/\phi} & \text{if } \phi < 0 \end{cases}$$

Thus whether a CES production function satisfies Inada conditions or not depends critically on the sign of  $\phi$ , that is, on the size of the elasticity of substitution. Therefore, whether the choice of a CES function itself imposes an upper limit on the size of technical progress and population growth or whether it displays endogenous growth of the economy in the steady state also depends on the sign of the parameter  $\phi$ . This point may be appreciated better with the help of figure 3.

Figure 3 show the two cases in which one of the Inada conditions could be violated by the CES production function. If  $\phi < 0$ , that is if  $\sigma > 1$ , then we are in the upper part of figure 3. An interior steady state exists provided the population growth rate, such as  $n_1$  in figure 3, satisfies the condition

$$(11a) \quad n > s(1-a)^{-1/\phi} - \delta.$$

However, if the growth rate of the labor force is less than  $s(1-a)^{-1/\phi} - \delta$ , such as  $n_2$  then the economy will have an endogenous growth rate of  $[s(1-a)^{-1/\phi} - (n + \delta)]$  at the steady state. Similarly, if  $\phi > 0$ , that is if  $\sigma < 1$ , the lower part of the figure shows that to have a nontrivial steady state, the growth rate of the effective labor force, such as  $n_2$  in figure 3, should satisfy the condition

$$(11b) \quad n < s(1-a)^{-1/\phi} - \delta .$$

These results were initially obtained by Pitchford (1960) and evaluated for its empirical relevance by Sato (1963). This condition imposes a limit on what growth rates on the effective labor force can be applied if we wish to guarantee a steady state solution of the economic system given CES production functions.

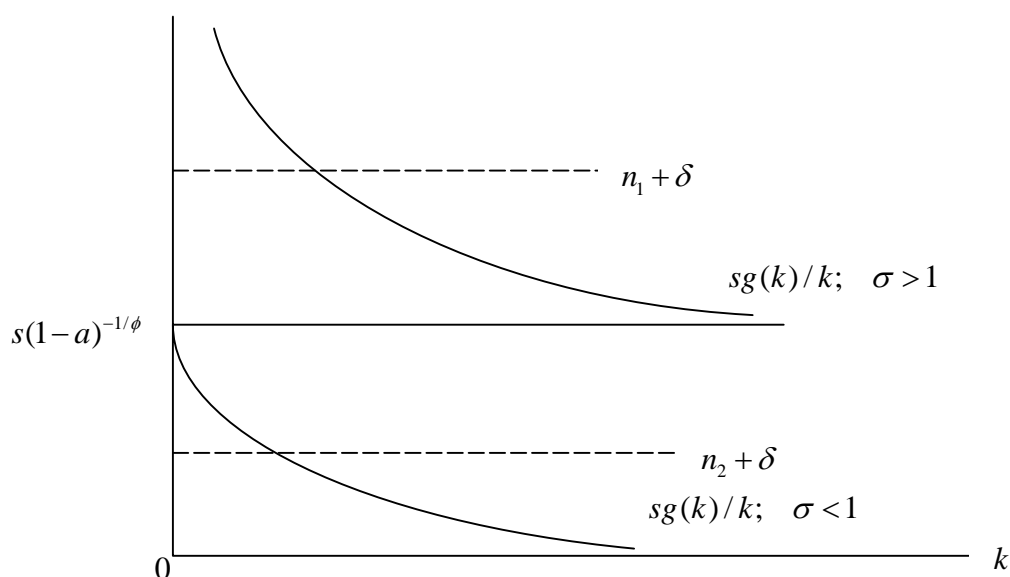


Figure 3: CES production functions and Inada conditions

Most CGE models maintain that the primary factor nest of the production function is characterised by a CES or Cobb-Douglas function or a nest of both functions. As it is important to make sure that condition (11b) is not violated, it is not difficult to understand why in CGE models the elasticities of substitution between primary factors are normally maintained equal to or greater than unity.<sup>1</sup> Condition (11b) would certainly

---

<sup>1</sup> MIT's model EPPA assumes the value of the elasticity of factor substitution between L and K to be unity in all but the nuclear electric sector, for which it is 0.5 (See Table 2 in Babiker, *et al.* 2001). The

be violated if  $\phi > 0$  is coupled with a sufficiently low savings rate, such as equal to or less than the depreciation rate.<sup>2</sup>

#### **4 Benge-Wells extension of the Solow-Swan growth model for a small open economy**

The Solow-Swan model of a closed economy is formally extended for a small open economy in Benge and Wells (2002). Since a small open economy can borrow from or lend to the world capital market at a given interest rate, it does not have to maintain an equality between domestic savings and domestic investment. It can continue to accumulate foreign debt/assets even in the steady state. The necessary modification of the above model required to accommodate smallness and openness can be summarised in the following terms.

We describe the production function and effective labor growth process by equation (1) and (2) respectively and the intensive production function by equation (3). Assuming perfect capital mobility, the equilibrium capital stock can be obtained from the profit maximising condition:

$$(12) \quad g'(k) = \rho + \delta$$

where  $\rho$  is the *exogenous* global interest rate and the prime denotes the first derivative. Equation (12) implies the satisfaction of the arbitrage condition between foreign bonds and domestic physical capital, whose rate of return is equal to the rental rate less the depreciation rate. Profit maximisation implies that the rental rate would be equal to the rental rate for capital.

Gross investment,  $I$ , satisfies

$$(13) \quad I = dK / dt + \delta K$$

which means that the changes in the capital stock is equal to net investment.

In per capita terms equation (13) can be written as:

GTAP5 data base maintains the elasticity of substitution at 0.24 for all primary agricultural, coal, oil, natural gas and minerals n.e.c. sectors and maintains values greater than unity for all other sectors. (see Table 20.2 in Dimananan, McDougall and Hertel, 2002). Similarly, WorldScan employs Cobb-Douglas functions to aggregate all primary factors in each sector (CPB, 1999). Note that Cobb-Douglas function satisfies Inada conditions.

<sup>2</sup> The GTAP5 data base does not account for foreign assets held by countries and thus ignores interest income from net foreign asset holdings. By doing so it over estimates the saving rates of the borrowing countries and underestimates the saving rates of the creditor countries. Currently the savings rates range from -14% for Mozambique to 48% for Malaysia. Central America and Tanzania are other regions whose average savings rates were below 4%. Of course aggregation can be used to hide such problems, but will resurface with appropriate disaggregation again.

$$(14) \quad i = dk / dt + (n + \delta)k .$$

As the domestic production (GDP) can differ from income (GNP) in an open economy, we write the income determination equation in per capita terms as

$$(15) \quad x = y + \rho f$$

where  $x$  is income, and  $f$  is foreign asset per effective worker respectively.

Defining  $w$  as the wealth per effective worker, we obtain

$$(16) \quad w = k + f .$$

Since by definition

$$(17) \quad dw / dt = (1 / L)dW / dt - nw$$

and the saving of the economy has to be allocated either to domestic gross investment or to foreign bonds we have

$$(18) \quad S = sX = dK / dt + \delta K + dF / dt = dW / dt + \delta K$$

From (18), (17) and (15) we can write

$$(19) \quad dw / dt = s(g(k) + \rho f) - nw - \delta k$$

Using equation (16) we can rewrite equation (19) as

$$(20) \quad dw / dt = [s(g(k) - \rho k) - \delta k] - (n - s\rho)w$$

Given that the optimal capital stock per person would be determined by equation (12), for given  $\rho$ , the term  $[s(g(k) - \rho k) - \delta k]$  in equation (20) is a constant. This equation can be integrated to solve for the time path of the wealth variable as:

$$(21) \quad w_t = \frac{[s(g(k^*) - \rho k^*) - \delta k^*]}{n - s\rho} + \left[ w_0 - \frac{[s(g(k^*) - \rho k^*) - \delta k^*]}{n - s\rho} \right] e^{-(n-s\rho)t}$$

It is clear from equation (21) that in order to have this solution stable, we must have

$$(22) \quad n - s\rho > 0 .$$

So that as  $t \rightarrow \infty$ ,  $w_t \rightarrow w^* = \left[ \frac{s\{g(k^*) - \rho k^*\} - \delta k^*}{n - s\rho} \right]$ , which follows from equation (20)

as the steady state solution for wealth per effective worker. Moreover, to have a positive value for  $w^*$ , it is also necessary that

$$(23) \quad s(g(k^*) - \rho k^*) > \delta k^*$$

which simply means that the saving rate times the wage rate (saving out of the wage rate per effective worker) be greater than the depreciation requirement for the ‘tools’ used by the worker.

Thus the steady state equilibrium of a small open economy is characterised by the conditions  $dw/dt = 0$  in equation (19) or (20) together with the condition  $dk/dt = 0$  in equation (14), which implies from equation (16) that  $df/dt = 0$ . Thus foreign asset holdings are not necessarily required to be depleted at the steady state of the open economy version of the Solow-Swan model. The level values of the state variables – the capital stock, the level of total wealth and total foreign assets – all grow at the exogenous growth rate of the effective labor supply keeping their per effective worker values constant over time. It is easy to see that total production, income, savings and investment all grow at the exogenous growth rate of the effective labor force.

It is nevertheless important to make sure that the growth rate of the effective labor force satisfies condition (22).

From this Benge and Wells small open economy extension of the Solow –Swan model we can derive some further results that can provide useful insights for the model of the global economy.

Rewriting (20) alternatively using (16) we get

$$(24) \quad dw/dt = sg(k) - (n + \delta)k - (n - s\rho)f .$$

Which means that we will have

$$(25) \quad dw/dt = 0 \text{ if } f = [sg(k) - (n + \delta)k]/(n - s\rho) .$$

Alternatively, the condition (25) for steady state can also be re written as

$$(26) \quad f = k \left[ \frac{sg(k)/k - (n + \delta)}{(n - s\rho)} \right] .$$

As can be seen from equation (8) the numerator inside the square brackets in condition (26) is precisely the expression for the growth rate of  $k$  in the closed economy version

of the Solow-Swan model. Condition (26) simply states that for given values of  $(n, \rho, s, \delta)$  the steady state of the small open economy can be obtained with any values for the pair  $(f, k)$  provided the pair  $(f, k)$  satisfies condition (26). In this case, however, the pair has a unique value forced by equation (12).

Recall from the results of the previous section that if the elasticity of factor substitution is less than unity and the savings rate is sufficiently low, then a closed economy may not have a non-trivial steady state equilibrium. Thus there is a possibility of a steady-state-speed-limit for some closed economies. Now we can ask the same question again for a small open economy – does a speed limit apply? Equations (25) and (26) can be used to infer an answer to this question as follows.

**PROPOSITION 1:** A small open economy will have a steady state equilibrium for all growth rates of the exogenous factor (physical and productivity growth of labor in this case) provided condition (22) is satisfied. That is, condition (11b) is irrelevant for a small open economy. Therefore, theoretically, a small open economy will have no speed limit on the steady state (or balanced) growth rate.

**PROPOSITION 2:** In the steady state equilibrium whether a small open economy becomes a net borrower or is debt free or becomes a net creditor depends on whether

$$sg(k^*)/k^* \begin{matrix} < \\ > \end{matrix} n + \delta$$

Provided condition (22) is satisfied.

**COROLLARY 2.1** If a small open economy has  $sg(k)/k < n + \delta$  for all  $k$ , and condition (22) for dynamic stability is satisfied, then the economy becomes a net borrower in the steady state equilibrium (for all possible values of the global interest rate).

Proofs of both propositions and the corollary follow directly from equations (25) and (26).

## **5 Conditions for convergence to a steady state in intertemporal GTEM: lessons from Swan-Solow models**

As mentioned earlier, intertemporal GTEM is multi-sectoral and multi-country neoclassical growth model of the world economy. Each region (country or country group) in the model is behaving as if it is a small open economy while making choices. Hence most of the results obtained in the previous section are directly relevant to GTEM. However, there are some differences that need to be noted.

First, GTEM is multi-sectoral, while the above growth models have only one sector. This means that we need to consider any effects of the differences in the production functions for different sectors and thus pay attention to possible effects of any differences in the elasticities of factor substitution. It may be possible that some sectors

have elasticities of factor substitution greater than unity while elasticities of substitution in some other sectors may be less than unity. Hence the above conditions, such as (11b) have to be read as the one specified for some convex combination of sectoral elasticities of factor substitution.

Second, because GTEM is a multi-country model of the world economy, global savings must be equal to global investments in all periods. That is, the world economy itself is closed. The global interest rate is endogenously determined. Therefore the condition used to solve equation (20) has been violated; the solution of global models, like GTEM, is not as straightforward as it was in the case of Benge and Wells' extension of the Solow-Swan model. The model loses its recursive nature in the sense that the steady state value of per capita capital,  $k$ , cannot be determined from the production function alone before the model determines the global interest rate. The model becomes a simultaneous system of non-linear equations and most likely needs numerical solution.

Third, the multi-country nature also imposes some accounting restrictions on the model solutions. One such condition is that foreign assets held by all countries taken together should sum to zero - someone's debt is someone else's asset. This means, in a balanced growth path the weighted sum of per capita holdings of foreign assets should sum to zero. The weights in this case are the regional shares of effective labor units in its global supply in the base year. (Here labor units are compared on a one-to-one basis between regions. This is applicable only for the balanced growth path otherwise the weights will change).

Fourth, all production processes in GTEM require the use of intermediate inputs. Use of primary factors is only a part of the story. Intermediate inputs are produced domestically as well as imported. Therefore, the inter-industry linkages (both forward and backward linkages between industries located in different regions) play a crucial role. Whether additional commodities produced in a region can be absorbed by foreign entities and whether the foreign entities are growing enough to supply the required intermediate inputs during the growth process becomes an important question. Unless all regions are growing uniformly along a globally balanced growth path the global inter-dependence implies that even for a small open economy, the Solow-Swan results may not hold in a strict sense.

Fifth, capital stock adjusts instantaneously in the standard Solow-Swan model while in GTEM adjustments in capital stocks are made through changes in investment, which respond to changes in the relative rates of return. Inter-regional movement of installed capital stock is not allowed.

Now we state some additional results keeping these differences between GTEM and a standard Solow-Swan model in mind. We will continue to ignore these differences until we simulate the model in section 7.

**PROPOSITION 3:** If in a solution trajectory all regions of the world simultaneously satisfy either of the conditions:

- (i)  $sg(k)/k < (n + \delta)$  for all  $k$ , or
- (ii)  $sg(k)/k > (n + \delta)$

at all times then the trajectory cannot contain the steady state of the global economy. In a steady state equilibrium if one region satisfies condition (i), then there must be at least one other region satisfying condition (ii).

Proof: It follows from condition (26) that if the hypotheses of the proposition are satisfied, then  $f_t < 0$  for all regions (regional index suppressed). As equilibrium requires the weighted sum of net foreign asset holdings of all regions be equal zero at all times, equilibrium of the world economy is impossible with  $f_t < 0$  for all regions. The same argument applies to the case when all regions satisfy condition (ii) and obtain  $f_t > 0$  simultaneously. This means that if one obtains non-zero levels of net global debt or global assets at the finite terminal time in the simulation results then it should be taken as an indication of model divergence from the steady state path.

It is, however, worth noting at this point that if  $sg(k)/k = (n + \delta)$  holds for all regions, then the world economy will have steady state equilibrium and that will be identical to the closed economy version of the Solow-Swan model for each region.

It follows immediately, as an implication of proposition 3, that  $n > sg(k)/k - \delta$  simultaneously for all countries at the steady state is impossible. This impossibility, which was derived as condition (11b) in the case of CES production function, deemed irrelevant for a small open economy, can now be seen to be of some importance in a model of the global economy. If all regions of the world economy violate the condition, the world economy, which is a closed economic system, also violates it. Note that for this situation to be relevant we need the elasticity of factor substitution to be less than unity for all regions and/or the saving rate must be very small.

**PROPOSITION 4:** The momentum simulation of the world economy (that is, with zero growth rates of the effective labor force for all regions) will necessarily diverge from its steady state path.

Proof: The proposition follows directly from condition (22). This result holds in a model of a small open economy as well as in a model of the world economy with open regions.

The implication of condition (22) is that if the world economy has to have a steady state equilibrium then it must have a minimum growth rate of the effective units of labor force satisfying condition (22). This sets the lower bound on the exogenous growth rate of the effective labor force. Naturally a question arises: is there an upper bound (on the uniform growth rate of the exogenous factor), especially when the elasticities of factor substitution are greater than unity? We attempt to answer this question heuristically.

First, note that in a model of the world economy, the global interest rate,  $\rho$ , is an endogenous variable. Hence,  $\rho$  may take different values for different periods along the transitional path. Let  $\rho_t$  be the equilibrium value of the global interest rate that clears the global saving investment market at time  $t$ .

In this case equation (20), which describes the dynamics of the state variable – per capita wealth in each region – is no longer a differential equation with a constant coefficient and a constant term. It is a differential equation with a variable coefficient and a variable term. Therefore equation (21) does not represent a solution to equation (20). Whether the equation can be integrated analytically depends on the functional forms, the general form of the solution, however, would be

$$(27) \quad w(t) = e^{-\int_0^t (n-s\rho_v)dv} \left\{ A + \int [s(g(k_t) - \rho_t k_t) - \delta k_t] e^{\int_0^t (n-s\rho_v)dv} dt \right\}.$$

where  $A$  is some constant term.

Whether the general solution representing the transitional dynamics vanishes over a long period of time or not again depends on whether or not:

$$(28) \quad \int_0^t (n - s\rho_v)dv > 0.$$

So that

$$(29) \quad \lim_{t \rightarrow \infty} A \exp\left\{-\int_0^t (n - s\rho_v)dv\right\} = 0.$$

Satisfaction of condition (28) critically depends on the initial value of  $\rho_t$ ,  $n$  and  $s$  and whether  $\rho_t$  responds to changes in  $n$  or not. If it does not, then condition (28) reduces to condition (22) and equation (27) reduces to equation (21). But if it does, then satisfaction of condition (28) becomes critical to whether we will have a steady state for all reasonable values of  $n$ . Now we examine the behavior of  $\rho_t$  in three different cases:

First, we can argue that *an increase* in the *growth rate* of labor supply (and productivity growth) lowers the wage rate of the effective unit of labor and raises the rate of return on capital, which in turn raises investment demand, *ceteris paribus*. But in each period, the rise in income as a result of an increased growth rate of the effective labor supply will be proportionately lower, as diminishing returns set in since capital stock is built with increased investment in the past period than by growth in labor supply. This is most likely the case if the elasticity of substitution between the exogenously growing factor (labor) and the endogenously growing factors (capital) is near zero. As a result, growth rates of global savings will become gradually smaller with higher and higher growth rates of labor supply. In this case, it is possible that the growth rate of the global demand for investible funds overtakes the growth rate of the global supply of the funds;

the global interest rate  $\rho_t$  will eventually start rising with further increases in the growth rate of the effective labor supply. We can thus expect that  $s\rho_t$  exceeds  $n$  for sufficiently large values of  $n$ .

The above point is illustrated in figure 4 in which the 45° line OA graphs possible growth rates,  $n$ , of the effective labor supply measured along the horizontal axis. The curve labelled  $s\rho_t(n)$  graphs the global interest rate adjusted by the saving rate as an increasing and convex function of  $n$ . If the global interest rate behaves as illustrated in figure 3, then it is clear that the dynamic stability can be assured only for those  $n$  such that  $n_1 < n < n_2$ . Thus  $n_2$  defines the possible speed limit for the steady state growth rate.

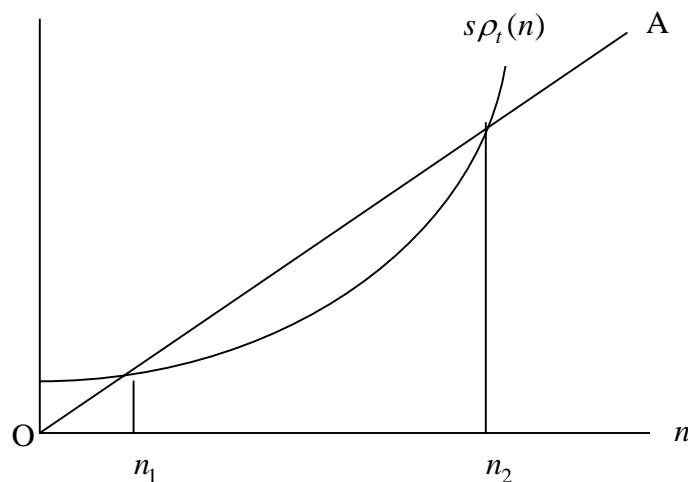


Figure 4: Behavior of the global interest rate with respect to labor supply changes

In the second case, we can argue the other way to claim that an increased value of  $n$  causes a fall in the global interest rate. This possibility arises if labor (exogenously growing factors) is a near perfect substitute of capital (or the endogenously growing factors). In this case the wage rate need not fall by much and the rate of return in capital may not rise and thus the demand for investment funds may not rise as much as saving rise as a result of a rise in global production and income. In this case the global interest rate may actually fall with the increase in the growth rate of the effective labor supply

and this may cause a problem of the second type that violates the condition (22) or (28) if the transversality condition has been imposed in the model.<sup>3</sup>

Note that a fixed saving rate can be obtained from intertemporally optimising choices of households with static expectations (Pant, Tulpule and Fisher 2002). In order that the problem of intertemporal optimisation be well defined, has bounded present values of future labor and capital incomes, and has the transversality conditions satisfied, it is necessary that

$$(30) \quad \lim_{t \rightarrow \infty} \int_0^t (\rho_v - n) dv > 0.$$

Alternatively, if the global interest rate remains constant, that is if  $\rho_t = \rho$ , to have the intertemporal optimisation problem well defined we must have  $\rho - n > 0$ .<sup>4</sup> But if the growth rate of the effective labor supply causes a decline in the global rate of interest, then we may be able to find a critical  $n^*$  such that for all  $n > n^*$  we will have  $n > \rho_t(n)$  for large  $t$  and therefore condition (30) will be violated.

It is also possible that the global interest rate responds to changes in the growth rate of the effective labor force but it does so in such a way that neither the interest rate falls below the steady state growth rate of the labor force nor its product with the savings rate exceeds the steady state growth rate of the effective labor force. In this case there is no speed limit for the balanced growth for the global economy.

Which of the three cases will be applicable to the world economy as represented in GTEM is not something that can be pinned down *a priori*. Hence we will have to rely on numerical simulations of the model and examine whether there are speed limits to the global economy as well as to the regional economies.

## **6 Is there a theoretical speed limit for the global economy?**

From the discussion of the previous section it follows that although it is quite difficult to obtain a closed form solution of equation (20), and other related differential equations, for all regions of the global economy, we can hypothesise, however, that both very low and very high growth rates of the effective labor supply may not lead to steady state solutions of the global economy. Therefore, we suspect that there are speed limits for the balanced growth of the world economy. The limits can roughly be described by the condition:

---

<sup>3</sup> In GTEM simulations we do not impose the transversality condition, we expect it to be satisfied by the solution automatically.

<sup>4</sup> For details on this condition in the context of both the closed and open economy version of the Ramsey model see chapter 2 and 3 in Barro and Sala-i-Martin (1995). The interval for convergence to be obtained below is more relevant to the Ramsey model than to a Solow-Swan model as the later does not contain the transversality condition.

$$(31) \quad \rho > n > \bar{s} \rho.$$

Where  $\bar{s}$  must be taken as the maximum of the saving rates of all regions and  $n$  the population growth rate as well as labor saving productivity growth. If the global interest rate responds to the size of the shock (the growth rate of effective labor supply), be it a convex or a concave function of  $n$ , condition (31) should be read as:

$$(32) \quad \rho(n) > n > \bar{s} \rho(n)$$

In this case it is possible for sufficiently large  $n$  that the increase in the global interest rate leads to a violation of the right hand side inequality of (32). It is most likely the case if elasticities of factor substitution are very low. Alternatively, a fall in the global interest rate in response to an increased growth rate of effective labor supply may lead to a violation of the left hand side inequality of (32) if elasticities of factor substitution are sufficiently high.

It must, however, be noted that small deviations from these conditions in early periods of the transition is not important, but the conditions must be strictly adhered to before the economy heads toward its steady state equilibrium. Hence we hypothesise that there are speed limits for the global as well as regional economies. These limits cannot be specified *a priori* unless the function  $\rho_t(n)$  is known. We can, however, get a feel for the limits for the case of constant global interest rate  $\rho$ . For example, if  $\bar{s} = 0.3$  and  $\rho = 0.1$ , then we must have  $0.1 > n > 0.03$ . This means that the steady state growth rate must be between 3 and 10 per cent. Similarly, if  $\bar{s} = 0.2$  and  $\rho = 0.05$ , then we must have  $0.05 > n > 0.01$ . This means that the steady state growth rate must lie between 1 and 5 per cent. If the global interest rate responds to the growth rate of the effective labor force then the speed limits should be obtained from condition (32) subject to the elasticities of factor substitutions in various regions and industries.

## **7 Exploring speed limits through GTEM simulations**

In this section we design and report the results of some diagnostic simulations of intertemporal GTEM that are aimed at exploring whether or not there exist limits to steady state growth rates of the world and regional economies. Violation of conditions that guarantee a steady state certainly leads to various problems in model solution. Some such violations may lead to unbounded values for some variables, causing arithmetic overflow and thus to model crashes. Alternatively, the model may solve but the solution may violate basic assertions, such as non-negativity of level values of some variables or violations of some adding-up conditions such as the condition specifying that global debt should sum to zero. Some of the model crashes can be avoided by choosing the values of some parameters appropriately. For example, in a closed economy version of the Solow-Swan model if all elasticities of substitution were set to be greater than unity, that is - if all  $\phi < 0$ , then solution failures for this reason can be avoided. But whether this can be done or not remains as an empirical issue.

We first explore the limits to the steady state growth rates of the global economy. To do so, we shock the model with uniform growth rates of all factors of production, whose supplies are exogenously determined. Although the model can be solved in a year-on-year mode, to save time the model was run in 5-year time steps, making uniform growth assumption within each time step. The GTEM database was aggregated into four regions – OECD90 (all members of OECD at 1990), ALM (all regions in Africa, Latin America and Mexico), REF (all economies in transition) and ASIA – to match the aggregation used in the special reports in emission scenarios (SRES) of the IPCC. For this aggregation, the average saving rate in the 1997 (GTAP5 database) was found to be around 30 per cent and the global average rate of return at around 10 per cent. This implies speed limits for the global economy of about 3-10 per cent had the global interest rate remained fixed. But as mentioned earlier, the elasticities of factor substitution were small only for the agricultural sectors and for all other sectors in all regions, the values were greater than unity. Hence we have a very strong possibility of global interest rate falling with increases in the growth rates of the exogenous factor supplies.

**Balanced global growth path:**

We applied uniform growth rates of 0, 2 per cent, 3 per cent, 4 per cent, 5 per cent and 10 per cent for the supplies of land, natural resource, labor and population across all regions. The time paths of the growth rates of real GDP and GNP (levels, not per capita) are reported in the charts 1-6.

The simulation results show that the global economy converged to the steady state for all growth rates of effective labour supply (exogenous factors) between 2–5 per cent per annum. For simulations with shocks higher than 5 per cent the model converges to a level slightly lower than that implied by the shock. For example, for a shock of a uniform 10 per cent, the growth rate of real GNP converges to 9.5-9.7 per cent. As expected the model showed signs of divergence in momentum simulation, in which case the growth rate of labor and other factors were set to zero (see chart 1).

**Biased regional growth rates:**

In a closed economy, the Solow-Swan model predicts that the level values of all real variables will grow at the exogenously given growth rate of effective labor supply (exogenous factors). In this case, we know (Pitchford 1960) that there could be speed limits on the steady state growth rate if the elasticities of factor substitution are less than unity and saving rates were low or there could be endogenous growth at the steady state if the elasticity of substitution were greater than unity.

In the case of an open economy, this kind of speed limit was not a problem. But the problem surfaced in another way. It was that the growth rate of exogenously supplied factors should be within the interval set by the global rate of interest and the product of the saving rate and the global interest rate. This condition has to be satisfied by each region as well as the global economy as a whole.

Now we ask how would steady state growth rates of the global economy and regional economies respond if factor supplies in different regions grow at different rates? To examine this situation we ran 11 counter-factual simulations – 5 simulations assume that factor supplies in ASIA only grow at uniform rates of 1 per cent, 2 per cent, 3 per cent, 5 per cent and 10 per cent per annum, in the next four simulations we assume that factor supplies in all non-OECD regions only grow at 2 per cent, 3 per cent, 5 per cent and 10 per cent per annum respectively and in the last two simulation we assume that factor supplies in the OECD only grow at 5 per cent and 10 per cent per annum respectively.

Of the 5 Asian growth scenarios, only the 1 per cent and 2 per cent growth scenarios converged in which case Asia grew at the steady state growth rate and the growth rate of other regions tapered off to zero. For growth rates of greater than 3 per cent the model displayed signs of divergence and it crashed for a 10 per cent growth in exogenously supplied factors in Asia and no growth elsewhere. Results of simulating the 1 per cent, 2 per cent and 3 per cent rate of factor accumulation in Asian region on growth rates of regional real GDPs and GNPs are presented in charts 7-9 respectively.

Similarly, of four simulation with factor growth only in non-OECD regions at various rates, the 2 per cent growth rate in non-OECD region converged while the rest showed signs of divergence. The simulation with the 10 per cent growth in non-OECD factor supplies failed to solve. The 3 per cent and 5 per cent growth scenarios are reported in charts 10 and 11.

In chart 13 we have reported the implication of a factor accumulation only in OECD regions at the rate of 10 per cent per annum. The results show that while the growth rate of the OECD tends to converge to 10 per cent, the growth rates of real GNP in other regions do not taper off; they keep on growing and there is no sign of converging to a particular rate. We take this to represent a divergence from the steady state.

These results indicate that with regionally biased growth the global economy tends to begin to diverge at lower growth rates.

### **Biases in factor growth rates:**

In the third set of simulations we examine the implication of restrictions in the growth of some factors across all regions. We run four counter-factual simulations with the assumption that the supply of labor and population grow at 1, 2, 3 and 4 per cent per annum respectively while holding the supply of land and natural resources at their base year levels.

With original parameter values these simulations did not solve. We increased the elasticity of factor substitution for agricultural and for other sectors based on natural resources, for which the elasticities of factor substitution were less than unity, by 1. With this change all sectors had elasticities of factor substitution greater than unity. The model converged to the exogenous growth rate for the uniform shock of 1 per cent and 2 per cent, and diverged for 3 per cent and above. The path of real GDP and real GNP

of the regions for the 2 per cent and 3 per cent growth rates are reported in charts 13 and 14 respectively. The divergence was clear at a 4 per cent growth rate for labor supply. It can also be seen for the case of 3 per cent as reported in chart 14.

## 8 Conclusion

In this paper our major concern was to explore the possible restrictions that we may need to adhere to while generating baselines of the global economy for the next 100 years using the Global Trade and Environment Model (GTEM). The fundamental question raised at the outset was whether there is any limit to the size and nature of autonomous technical progress that could be assumed during the baseline simulation of the model.

As the current version of the model belongs to the class of Solow-Swan models we reviewed the theory behind these models and some possible restrictions on the nature of growth were inferred. More importantly it became quite clear that in a model of the global economy containing small and open regions with multiple sectors, the analytical solution of the Euler equation becomes infeasible, mainly because the global interest rate becomes an endogenous variable. A numerical solution method was employed to explore the questions raised.

It was observed that there is no upper limit, within reasonable bounds (say for example, less than uniformly 10 per cent per annum) to the size of technical progress if it is in a balanced growth path. That is, if all exogenous factors grow uniformly in all regions then the global economy and the regional economies would also grow at the same rate at the steady state equilibrium. This is basically the display of the real homogeneity property of the model. On the lower side, however, a divergence was observed if the uniform growth rate is less than a critical value defined by the product of the savings rate and the global interest rate.

In the real world, uniform growth in the supply of all factors (whether it is through physical growth or through productivity growth) in all regions is a near impossibility. A realistic description would be to assume some form of factor-biased and region-biased growth. To examine the convergence of the regional economies under these conditions a few factor-biased and region-biased simulations were run. The results clearly indicated that if elasticities of factor substitution were less than unity the factor-biased growth displays a speed limit very early. Even when the elasticities of substitution were made greater than unity the divergence of the global economy from the steady state path was observed at under a 3 per cent rate. In the case of the regionally biased growth, in which case the exogenous factors in some regions accumulate at higher rates than in others, we also observed divergence of regional economies in the sense that they did not appear to be converging to any particular growth rate. Although we would need further work to be specific on the limits to region-biased growth types, we can set 5 per cent as a reasonable upper bound for the steady state growth rate for any single region growing without having any exogenous factor accumulation in other regions.

Finally, the implication of this exercise is that if realistic economic growth rates for regional economies have speed limits then so will the growth rates of emissions. We need further work to be more specific about this proposition.

## References

1. Barro, R. and Sala-i-Martin, X. 1995, *Economic Growth*, McGraw-Hill, New York.
2. Benge, M and Wells G. 2002, 'Growth and current account in a small open economy', *Journal of Economic Education*, spring, 152-165.
3. CPB Netherlands Bureau of Economic Policy Analysis 1999, WorldScan, the core version, CPB, The Hague.
4. Dinaranan, B, McDougall, R. and Hertel, T. 2002, "Behavioral parameters" in Dimaranan, B. and McDougall, R. (eds) **Global trade, assistance, and production: the GTAP5 Data Base**, Centre for Global Trade Analysis, Purdue University.
5. Pant, Hom, Tulpule, Vivek, and Fisher, Brian S. 2002, "The Global Trade and Environment Model: A projection of non-steady state data using intertemporal GTEM" ABARE paper presented to Fifth Annual Conference on Global Economic Analysis, Grand Hotel, Taipei, Taiwan June.
6. Pitchford, J. 1960, 'Growth and the elasticity of factor substitution', *The Economic Record*, **36**, 491-504.
7. Pitchford, J. 2002, 'Trevor Swan's 1956 economic growth seminar and notes on growth', *The Economic Record*, **78**, 381-387.
8. Rogers, M. 2003, 'A survey of economic growth', *The Economic Record*, **79**, 112-135.
9. Sato, K. 1963, 'Growth and the elasticity of factor substitution: A comment – how implausible is imbalanced growth?', *The Economic Record*, **39**, 355-61.
10. Solow, R. M. 1956, 'A contribution to the theory of economic growth' *Quarterly Journal of Economics*, **70**, 65-94.
11. Swan, T. W. 1956, 'Economic growth and capital accumulation' *The Economic Record*, **32**, 334-61.

Chart 1: Growth rates of regional real GDPs and GNPs under momentum simulation

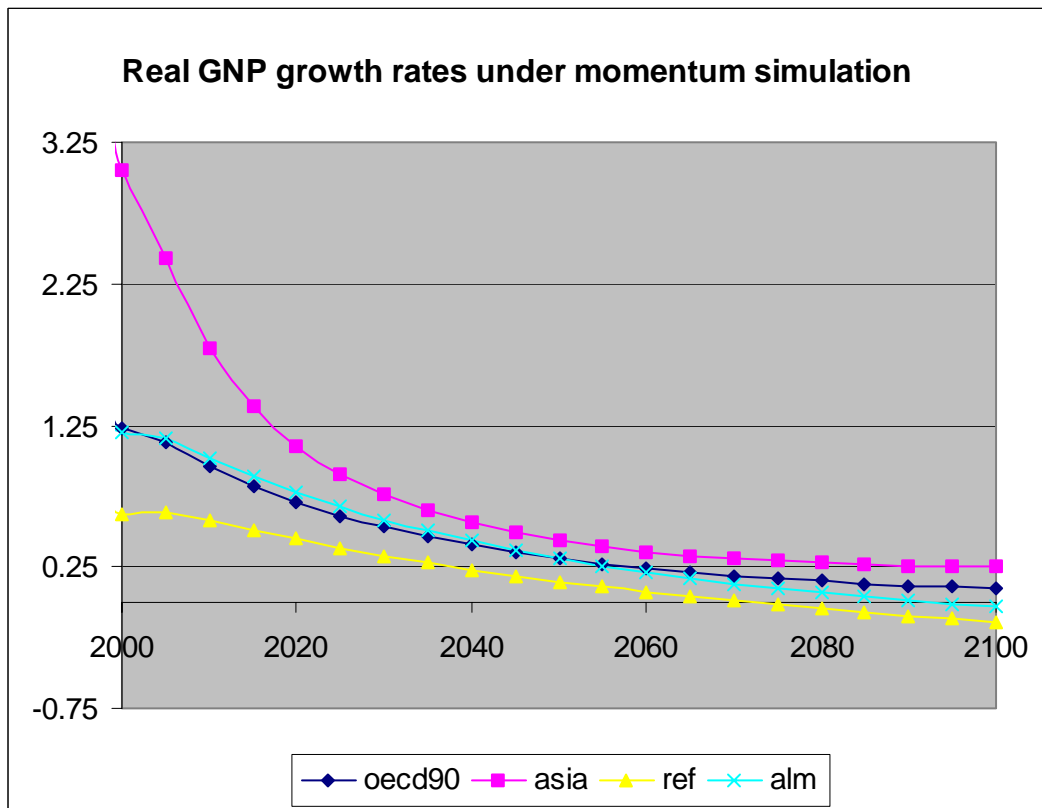
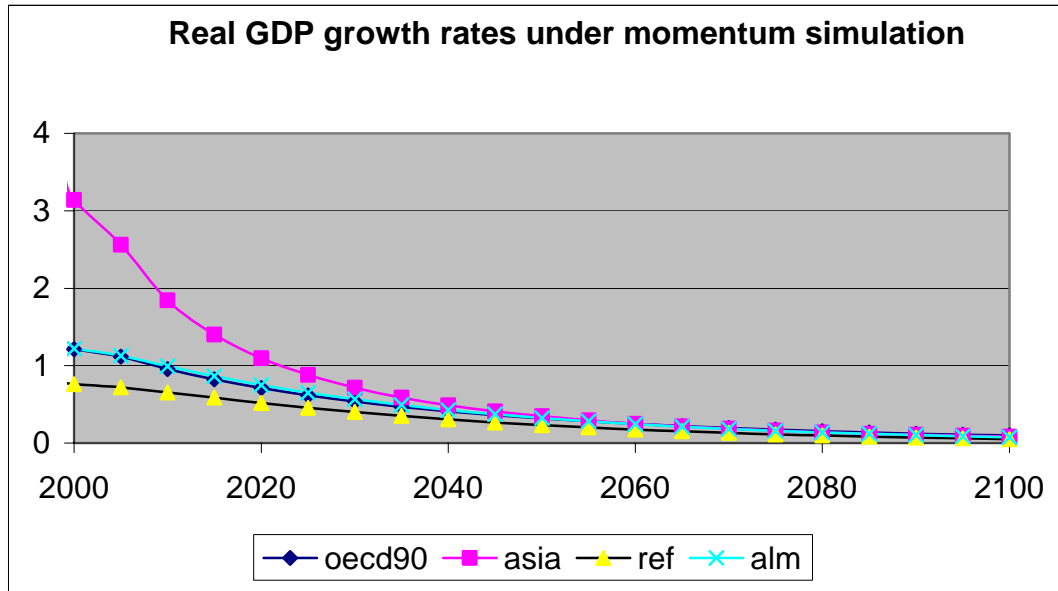


Chart 2: Real GDP and GNP growth rates under 2% pa balanced growth shocks

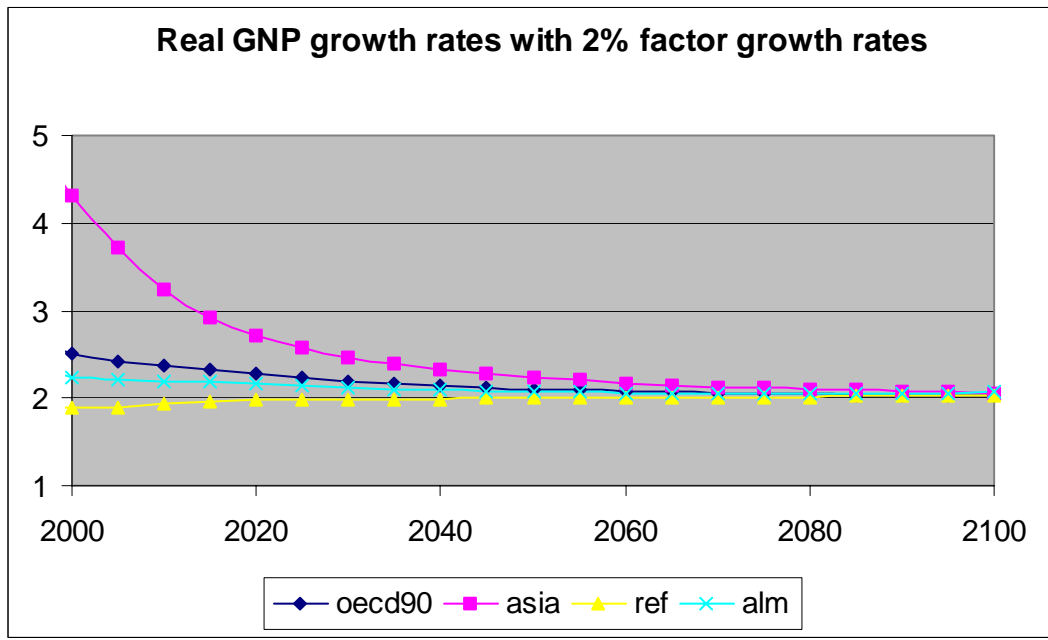
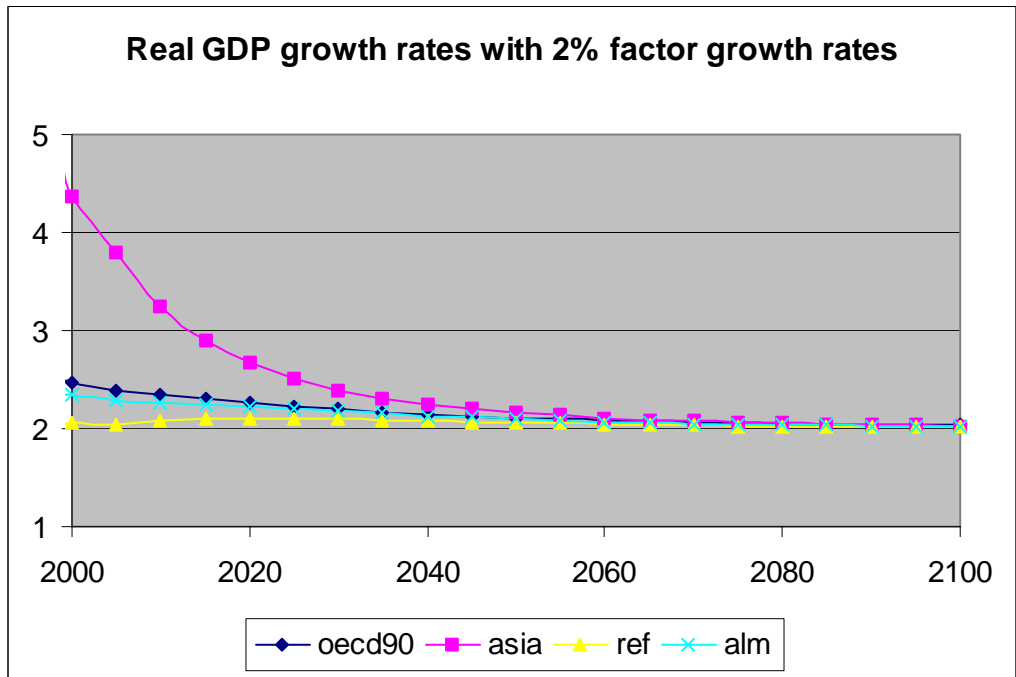
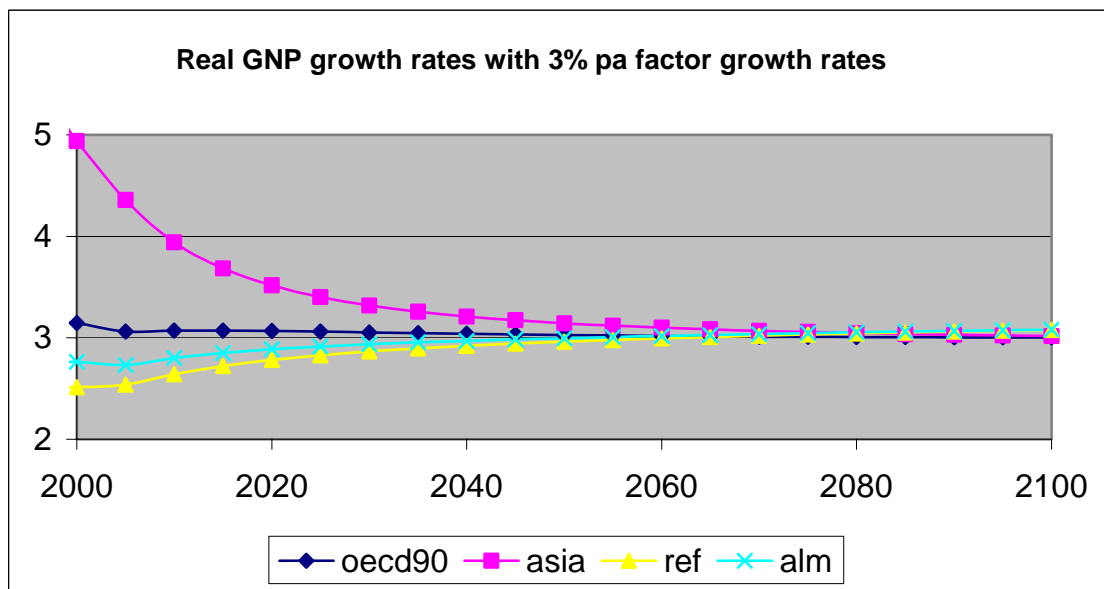
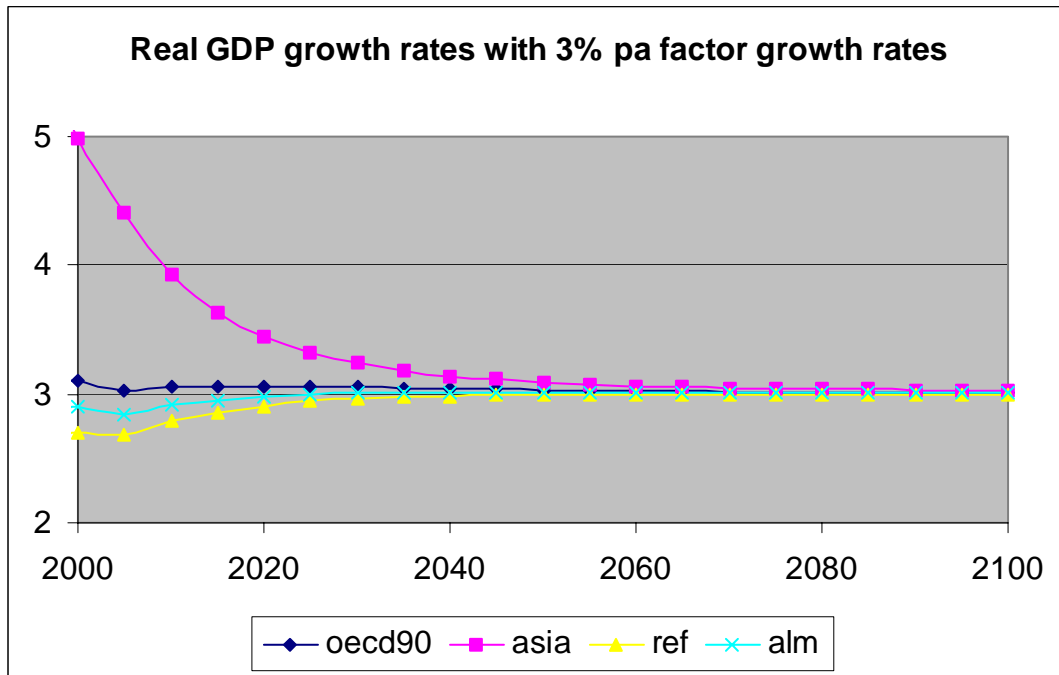


Chart 3: Real GDP and GNP growth rates with 3 per cent per annum factor growth shocks



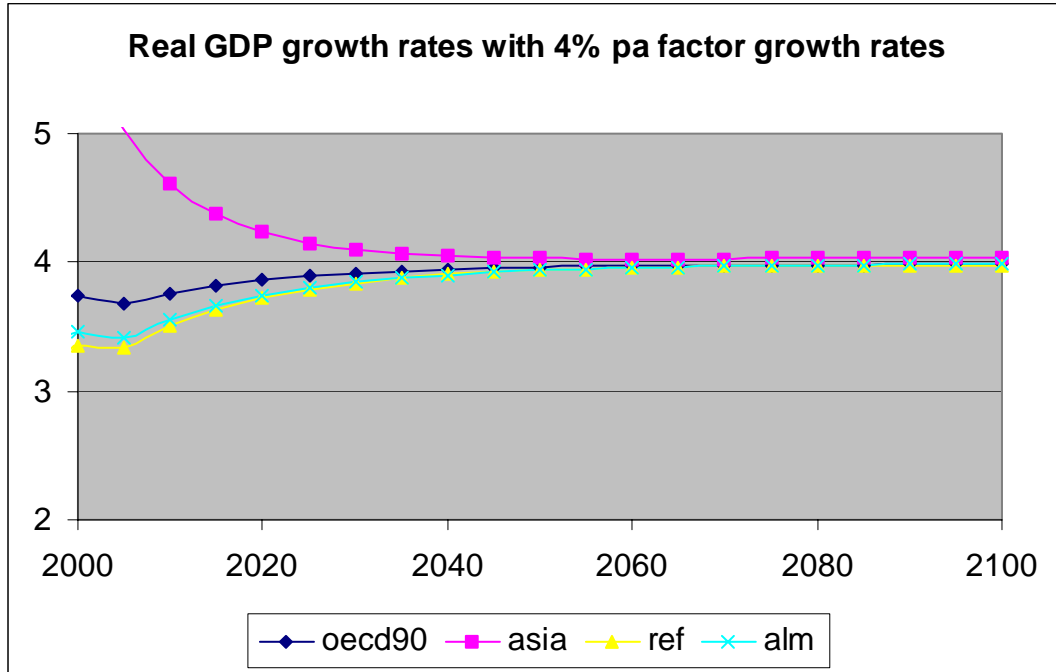


Figure 4: Real GDP and GNP growth rates under 4% factor accumulation

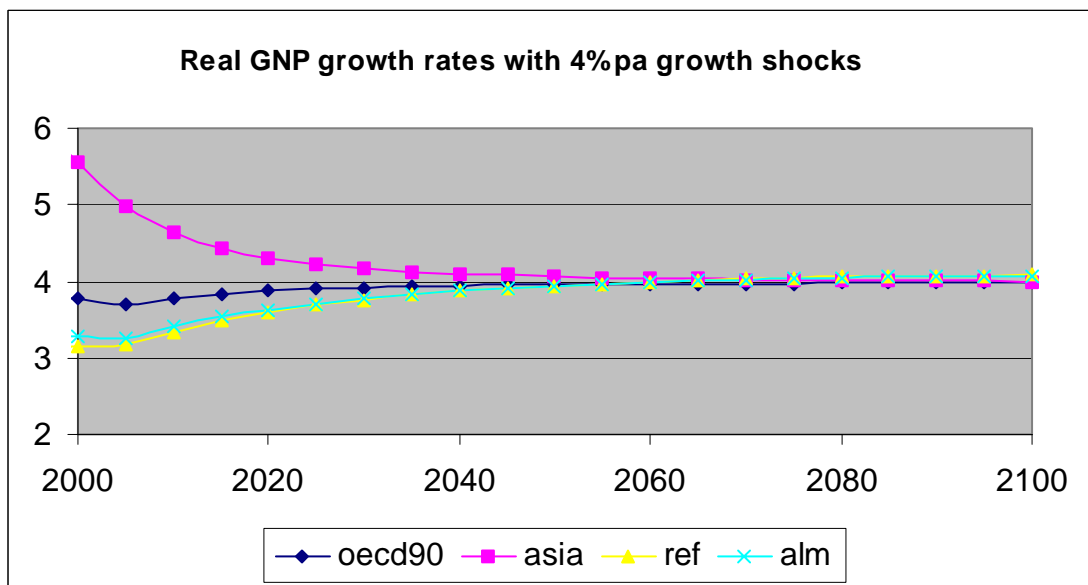


Chart 5: Real GDP and GNP growth rates with 5% pa factor accumulation

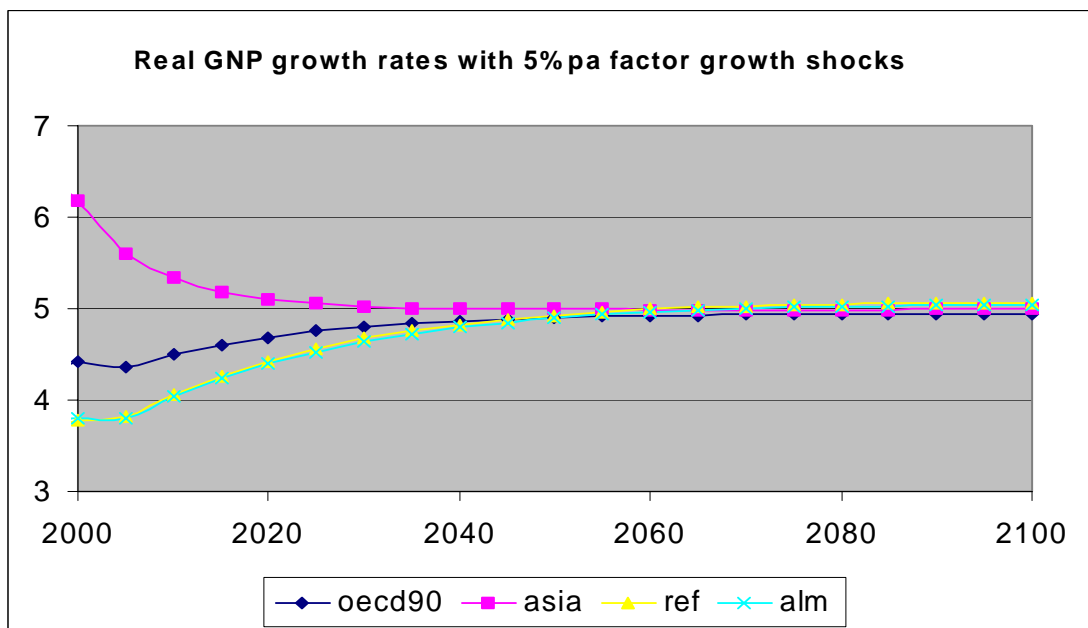
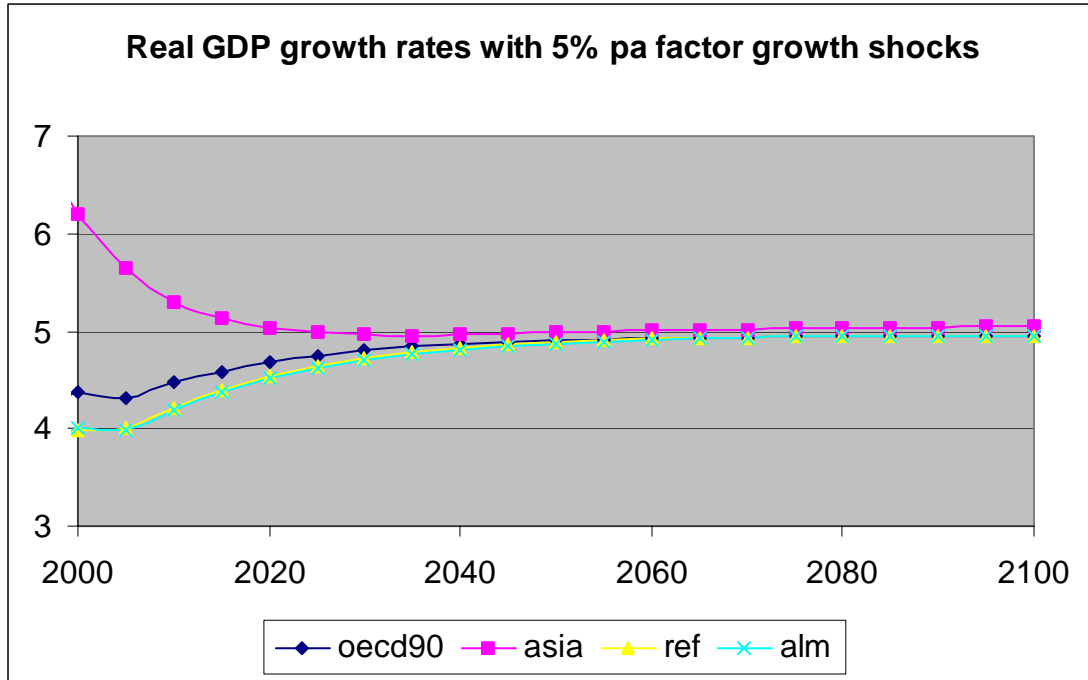
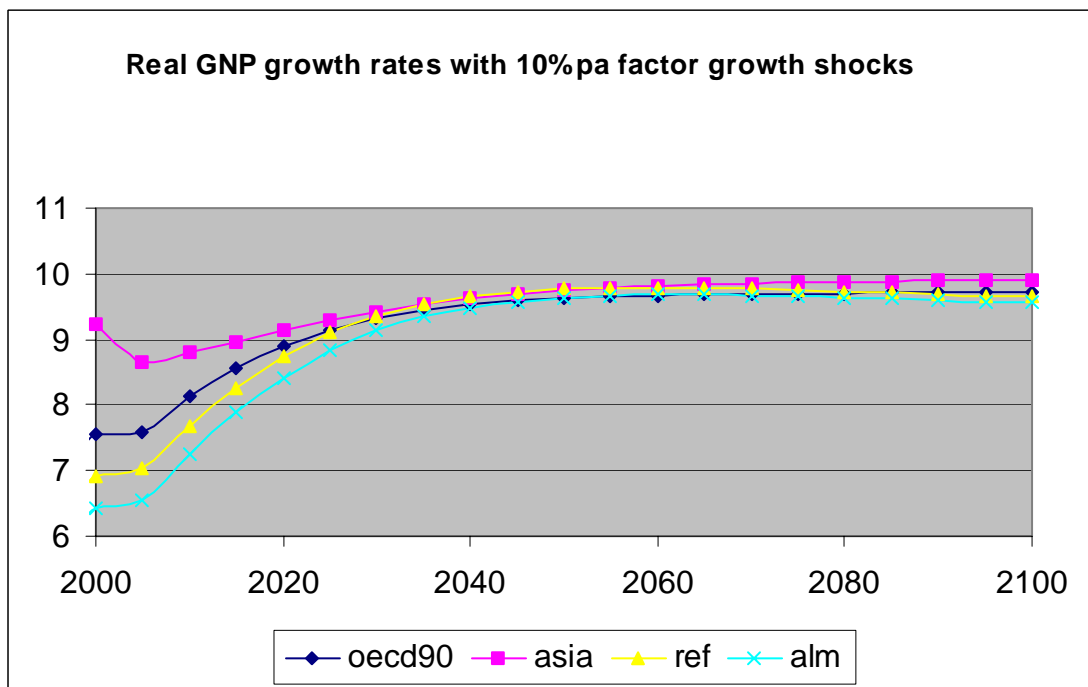
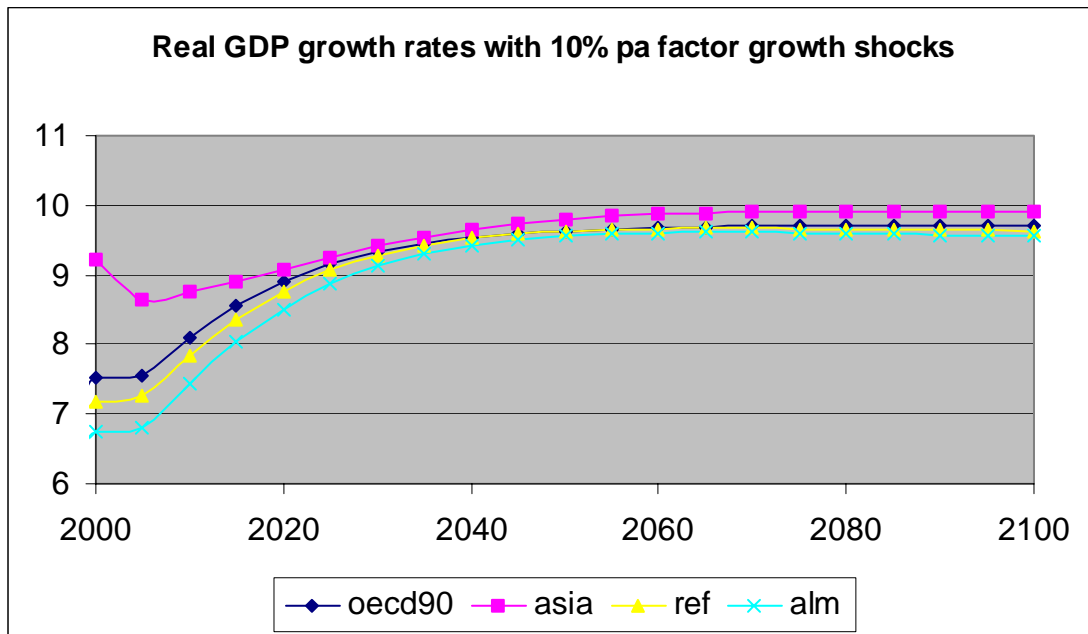


Chart 6: Real GDP and GNP with uniform 10% factor accumulation



Biased growth

Chart 7: Real GDP and GNP growth with 1% pa factor accumulation in Asia only

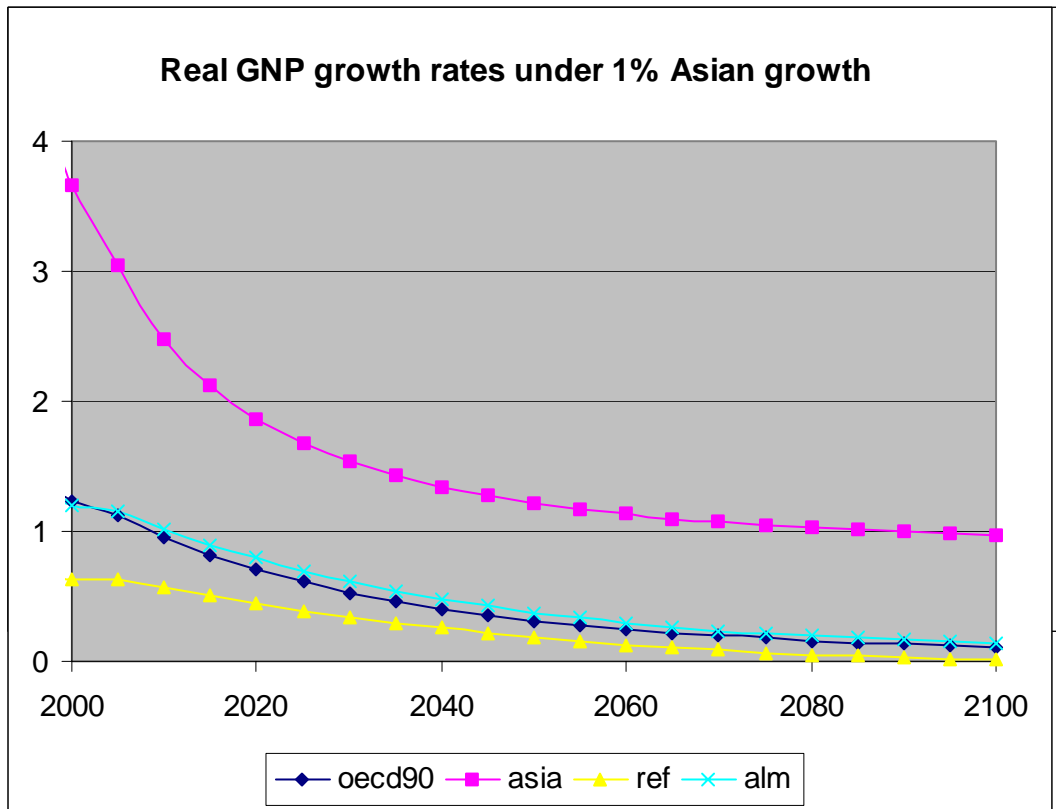
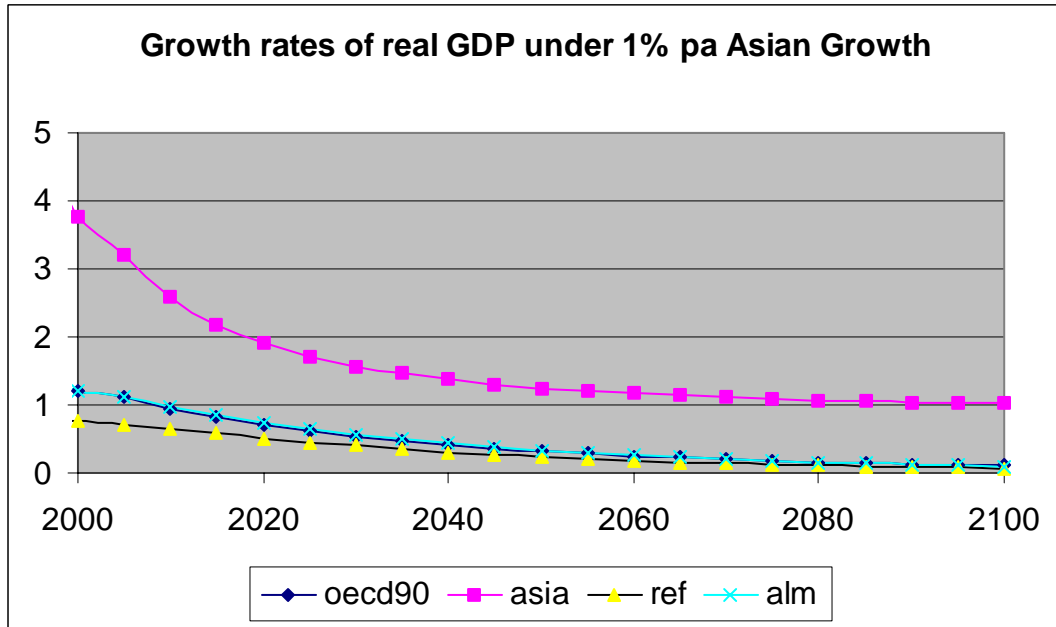


Chart 8: Real GDP and GNP growth with 2% pa factor accumulation in Asia only

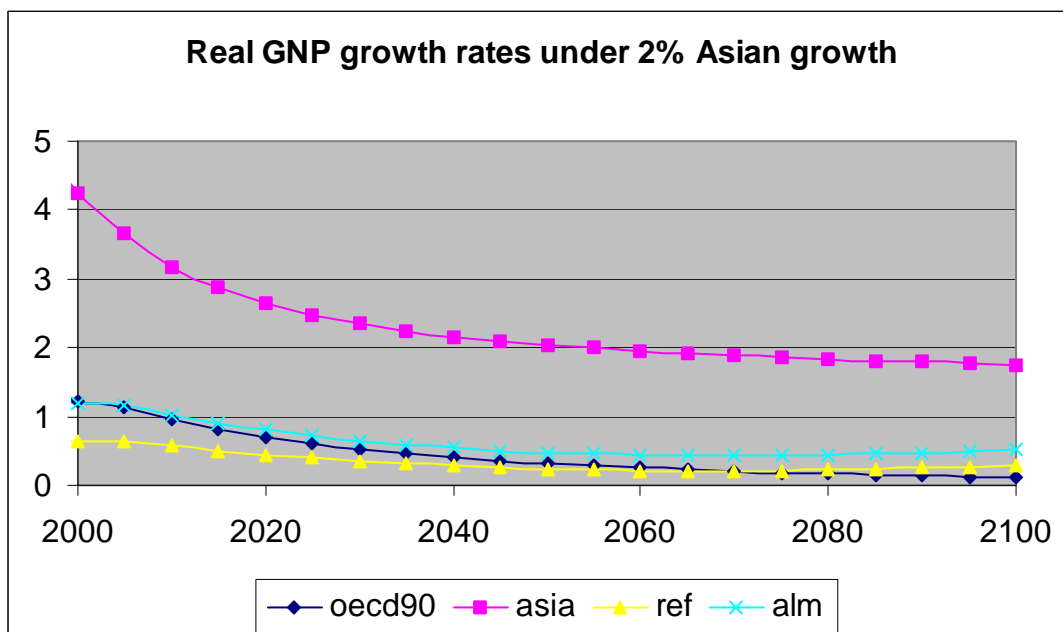
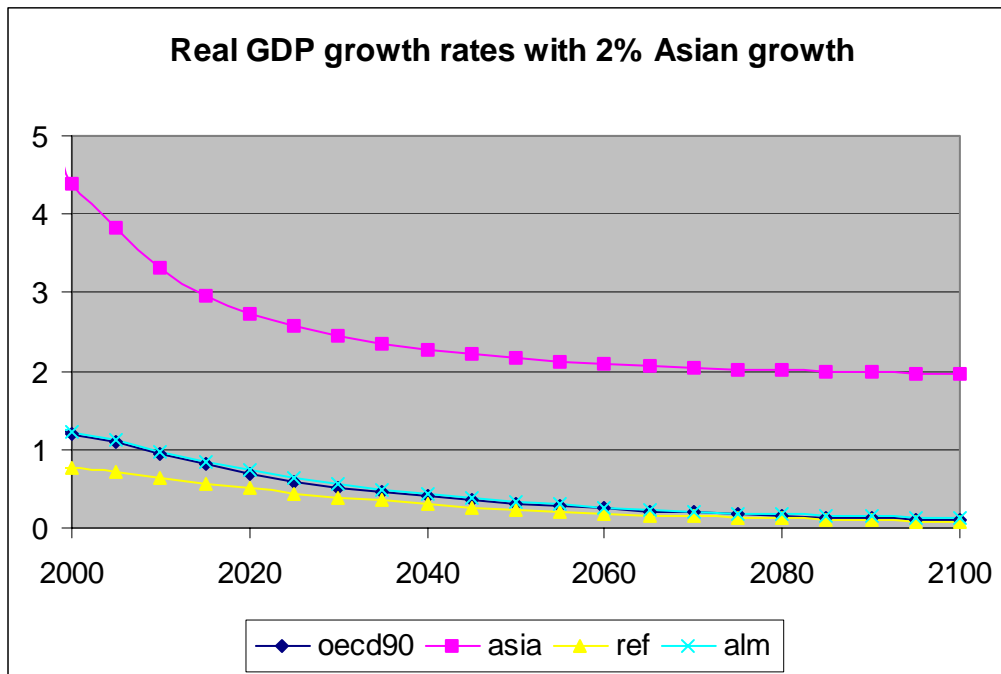
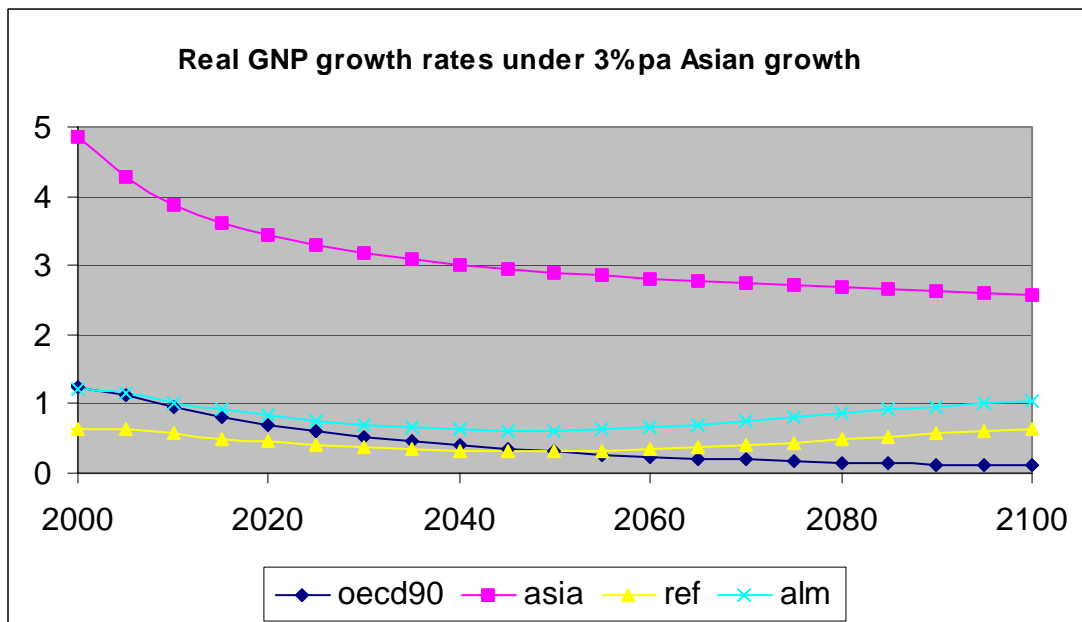
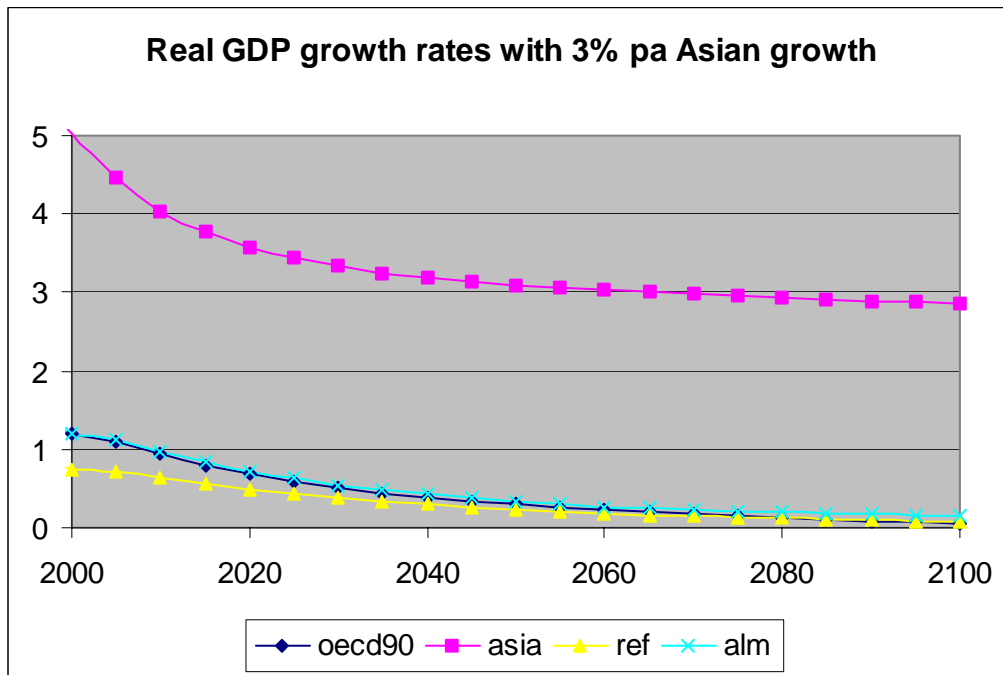


Chart 9: Real GDP and GNP growth rates with 3% factor accumulation in Asia only



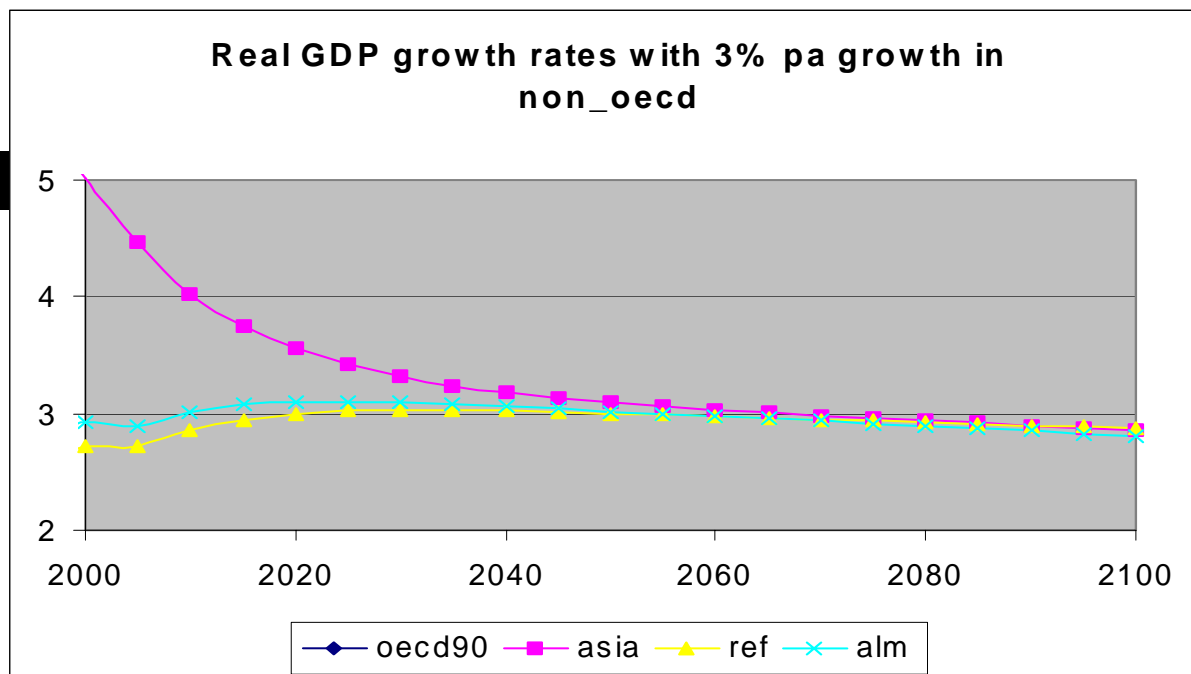


Chart10: Real GDP and GNP growth rates with 3% pa factor growth in all non-OECD regions

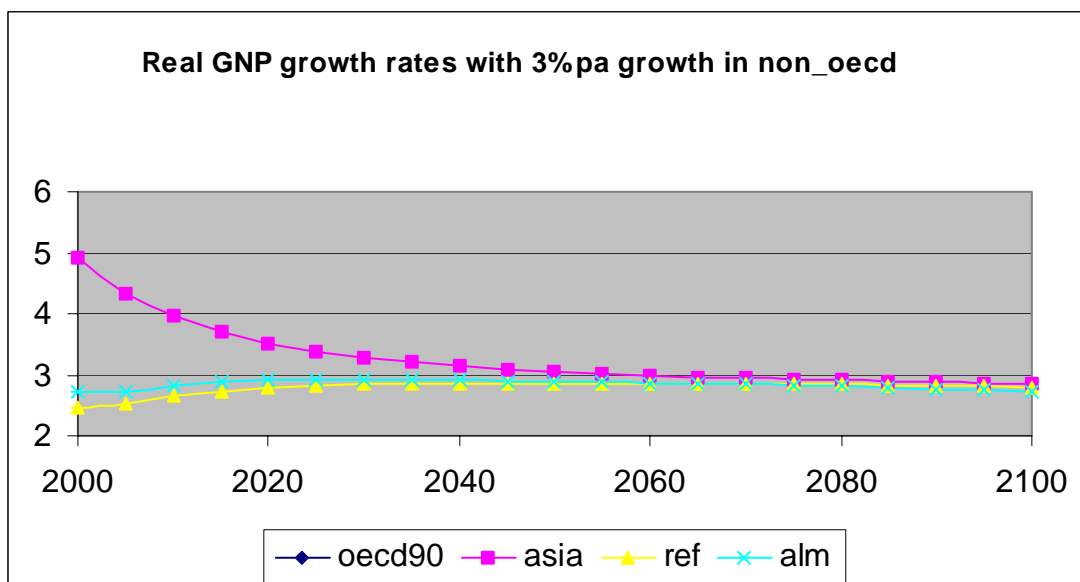


Chart 11: Real GDP and GNP growth rates with 5% pa factor accumulation in all non-OECD regions

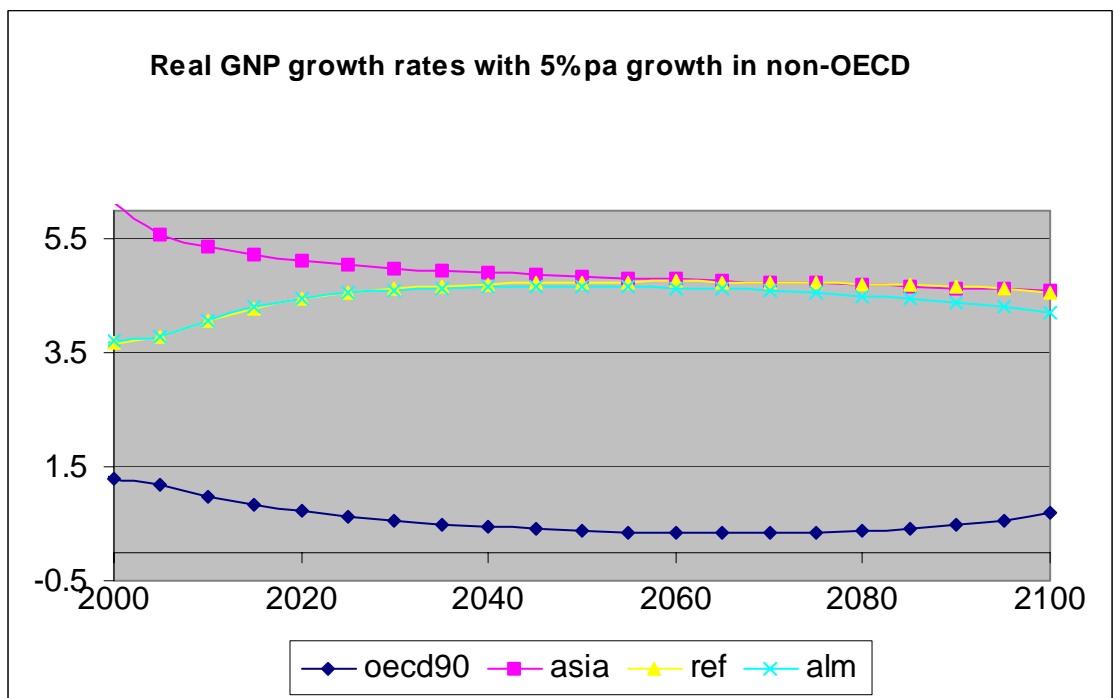
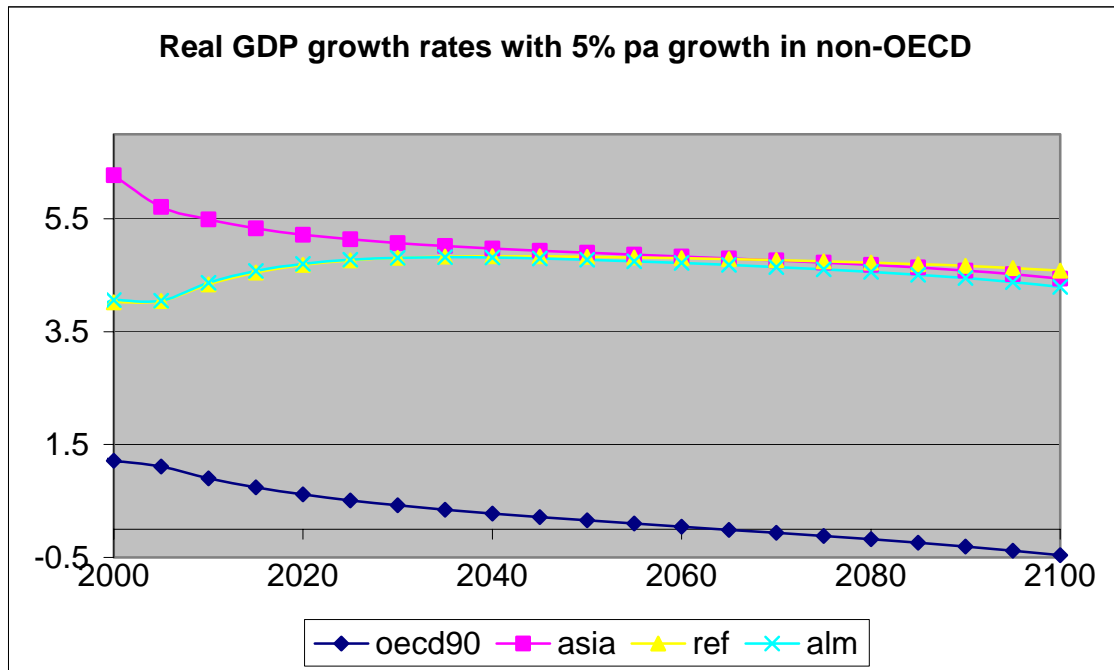
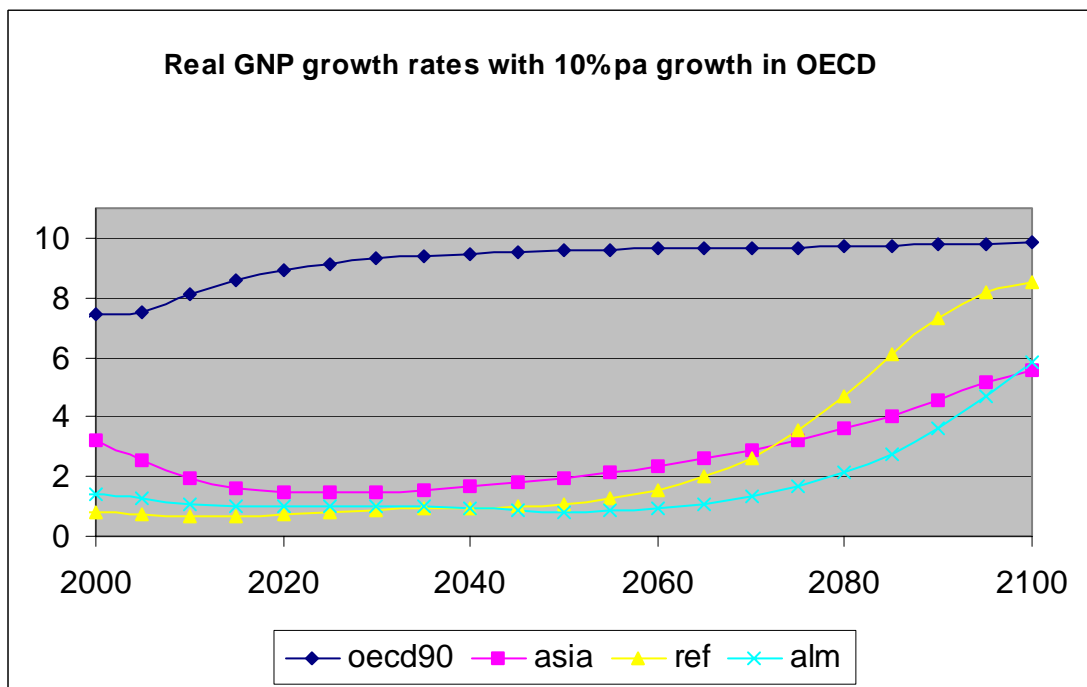
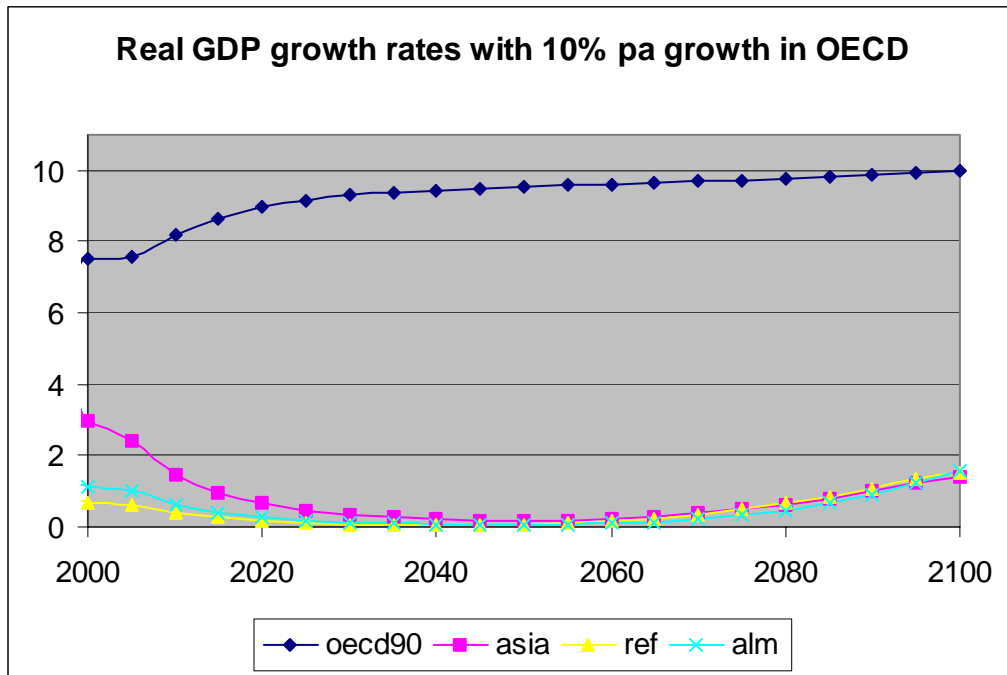


Chart 12: Real GDP and GNP growth rates with 10% pa factor accumulation in OECD regions only



FACTOR BIASED GROWTH: no growth in land and natural resources uniform growth in labor supply and population

Chart 13: Real; GDP and GNP growth rates with uniform labor supply growth at 2 per cent per annum

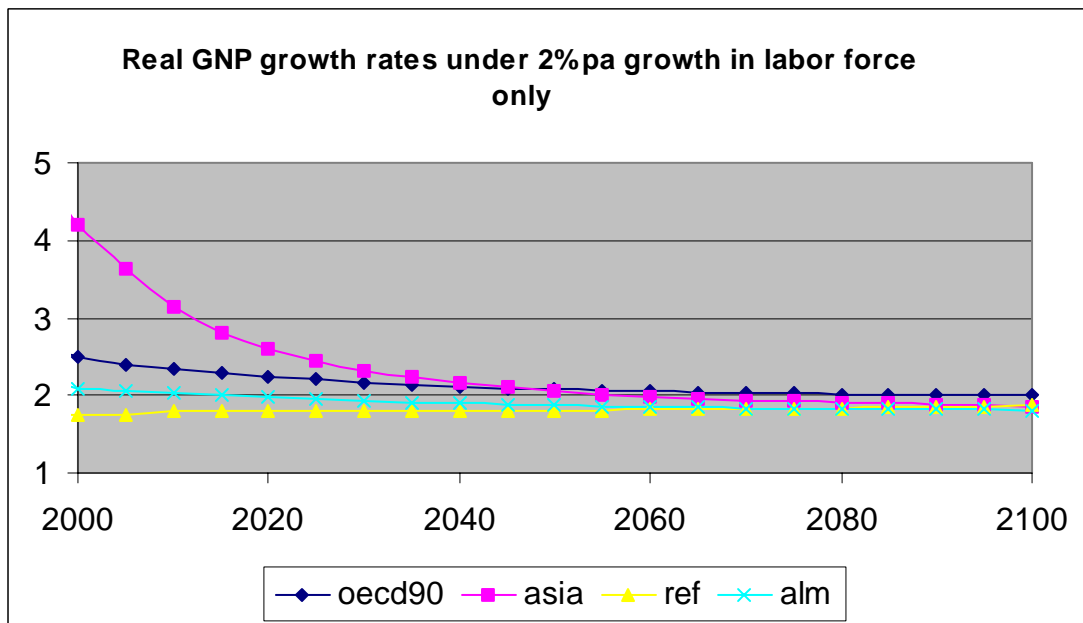
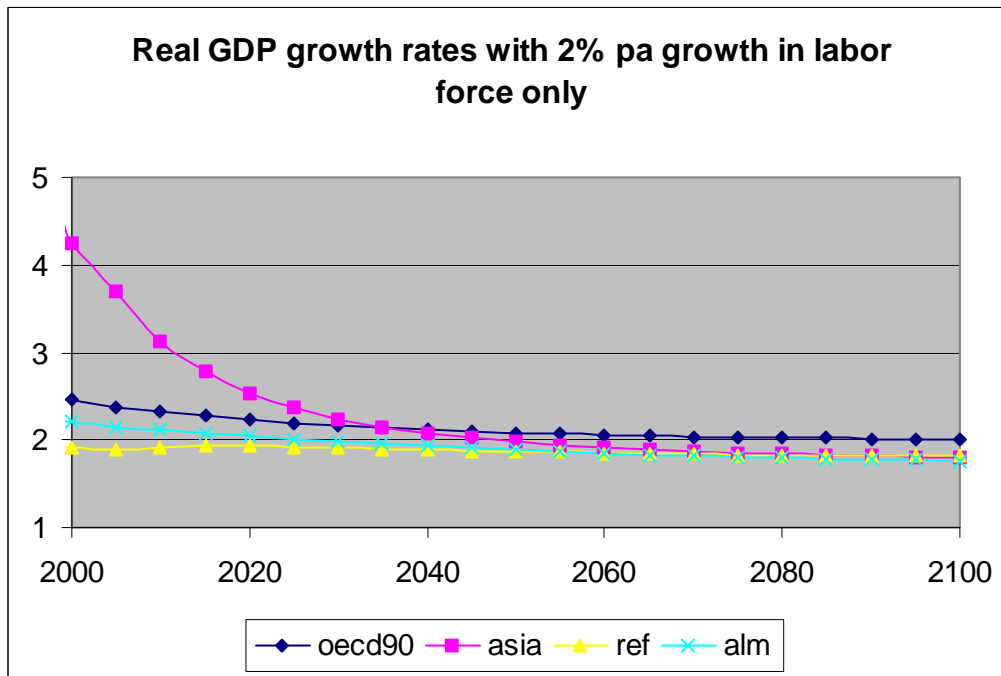


Chart 14: Real GDP and GNP growth rates with uniform 3% pa growth of labor and population only

