Economic analysis of risk and management of pest or disease incursions

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Pest and disease incursions can impose significant costs of forgone production and trade. Additionally, there are costs of management activities such as hazard reduction, eradication and control. The assessment of these costs and the determination of the optimal management response have attracted attention in recent years. Minimising the expected present value of the total of the above mentioned costs is one of the criteria for determining how to protect the economy from the effects of unmitigated hazards. It involves a tradeoff between additional expenditure on management activities such as hazard reduction and their expected benefits such as a reduced hazard rate of future incursions. In this paper, an analytical expression is developed for the expected discounted present value of all future incursion and management costs for the case of recurrent episodes of pest or disease incursion and eradication, where there is uncertainty about the arrival of disease incursions and about the outcome of the eradication process. Uncertainty about the outcome of the eradication process means that it is uncertain whether the pest or disease is completely removed at the end of the eradication process. The optimal expenditure on management activities for a risk neutral decision maker is obtained by minimising the derived expression with respect to the levels of management activities, subject to the relationships between management activities and their associated outcomes such as a reduction of the hazard rate. An example is provided to illustrate the determination of the combination of management activities that minimises the expected economic cost of pest or disease incursions.
Introduction

Pest and disease incursions have attracted increased attention in recent years. Economic analysis has been seen as essential to a full understanding of the problem of pest and disease incursions by providing a consistent and comprehensive assessment of the benefits and costs of — usually publicly funded — control alternatives (see, for example, Perrings et al. 2000, 2002; Evans 2003; Cao and Klijn 2004).

In this paper an economic analysis of management activities of pest or disease incursions characterised by recurrent episodes of incursion and eradication is put forward. Henceforth disease will be used to indicate either a pest or a disease. It is assumed that episodes of incursion and eradication are the outcome of a stochastic incursion process with a constant hazard rate per time unit. It is also assumed that there is uncertainty about the outcome of the eradication process after a known time — that is, there is a nonzero probability that eradication is incomplete at the end of the eradication process.

It is further assumed that initially either there is no disease or the disease is in an undetectable state and that each incursion results in an identical and constant flow of costs — including the eradication costs — until eradication is completed — that is, the time when either the disease is completely eradicated or the disease becomes undetectable. For a given hazard rate and a given probability of eradication, there is an expected discounted present value of the cost of recurrent episodes of incursion and eradication over all time. This expected cost also depends on the discount rate, the time required for eradication and the level of the constant flow of incursion costs.

Now assume that the hazard rate can be reduced by hazard reduction activity at an additional constant flow of hazard reduction costs and that the relationship between hazard rate and hazard reduction cost is known. The duration of the hazard reduction activity and the associated cost can be the time from eradication to incursion — that is, a stochastic variable dependent on the hazard rate, or the whole infinite time horizon. The expected total cost of incursion and hazard reduction then depends on hazard reduction activity. It is also assumed that the time required for eradication depends on the level of eradication activity — higher expenditure on eradication shortens the eradication time.

In this paper an analytical expression for the expected value of the discounted present value of costs of management activities and incursions is derived. For a risk neutral manager, the optimal level of management activities is where this expected value is smallest. For managers who are not risk neutral, the variance will also play a role in their decision making as they may, for example, trade off an increase in expected cost for a decrease in the variance of cost. An analytical expression for the variance can be
derived for some cases (for example, see Cao and Klijn 2004). For the case presented in this paper, however, derivation of an analytical expression for the variance has not been attempted.

Related earlier work is Jensen (2002), in which optimal hazard reduction is analysed dynamically for the case of a single incursion that is never to be eradicated; and Shogren (2000), in which a static approach is used to analyse optimal hazard reduction. Dynamic analysis of additional infections from across borders for a pest or disease that is already endemic here and is never to be eradicated can be found in Beare and Hinde (2001) and Leung et al. (2002). Also, Cao and Klijn (2004) discussed optimal hazard reduction for recurrent episodes of incursion and eradication.

Method
Assume that initially (at time \(t = 0\)) there is no disease with a probability \(q\), and a state of undetectable disease with a probability \(1-q\). Starting from no disease, an incursion can occur at any time with a probability per time unit (the hazard rate), \(p\), that is independent of time. The corresponding probability density of first incursion at time \(t\) is exponential, \(pe^{-pt}\). Starting from an initial undetectable disease state, the disease evolves and is detected in \(w\) units of time\(^1\). Hazard reduction is contemplated at a constant cost flow, \(h\). The hazard rate is assumed to be a monotonically declining function of the cost of hazard reduction, \(p = p(h)\).

An incursion is assumed to result in a given constant flow of costs, \(c\), that includes eradication costs \(c_e\) and production and trade loss costs \(c_l\), until the end of the eradication process \(T\) time units later. It is assumed that there is a known relationship between the eradication costs \(c_e\) and duration of the eradication process \(T\), \(T = T(c_e)\). An expression is then derived for the expected discounted present value of the sum of the costs of hazard reduction and incursion over an infinite time horizon as an explicit function of hazard reduction and eradication activities. For a risk neutral decision maker, the optimal level of hazard reduction is found when this expected cost is smallest. In box 1 the eradication process is explained in more detail.

In figure 1 the recurring process of disease incursion and eradication is shown. At the starting time \((t = 0)\), there is a probability \(q\) of being in a no disease state, and probability \(1-q\) of being in an undetectable diseased state.

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\(^1\) Note that surveillance measures can be used to detect the disease earlier (for example, see Kompas et al. 2003); however, it is not considered in this paper and can be easily added in the analysis.
(1) Starting from an initial state of no disease, there will be a first incursion at time $k_1$ with a probability $pe^{-pk_1}$. Then, the disease will be detected at $w$ time units later (that is, $t = k_1 + w$). There is a constant flow of cost $h$ associated with a hazard rate $p$ from $t = 0$ to $t = k_1 + w$, where $p = p(h)$. The eradication process starts at the time of detection ($t = k_1 + w$) and finishes $T$ time units later ($t = k_1 + w + T$). There is a constant cost $c = c_e + c_i$ during the eradication process (that is, from $t = k_1 + w$ to $t = k_1 + w + T$). The eradication time $T$ depends on eradication expenditure $c_e$ (that is, $T = T(c_e)$). At $t = k_1 + w + T$, the situation becomes the same as that at $t = 0$. This whole process continues indefinitely.

(2) Starting from an initial state of undetectable disease, the disease grows and is detected at $w$ time units later (that is, $t = w$). There is a constant flow of cost $h$ associated with the hazard rate $p$ from $t = 0$ to $t = w$. The eradication process starts at $t = w$ and ends $T$ time units later ($t = w + T$). There is a constant cost $c = c_e + c_i$ during the eradication period (that is, from $t = w$ to $t = w + T$). At $t = w + T$, the situation becomes the same as that at $t = 0$. This process again continues forever.

**Box 1: Explanation of the eradication process**

The eradication process involves an eradication activity with an associated cost and is aimed at removing the disease. The eradication time $T$ — that is, the period during which the eradication process is applied — is assumed known and given, and depends on the expenditure flow $c_e$ incurred during the eradication process — that is, $T = T(c_e)$. At the end of the eradication process, it is not certain whether the disease has been completely removed. It is assumed that the probability, $q$, of the disease being completely removed after the eradication process is known and given. This $q$ is assumed to be the same as the probability of the initial state (that is, at $t = 0$) being disease free. The state of incomplete removal after the eradication process, has probability $1-q$, and means that the disease has been reduced to an undetectable state.

The decision variables corresponding with the process shown in figure 1 are the hazard reduction cost, $h$, and the eradication cost, $c_e$. The cost of $h$ is in the first instance assumed to be positive and constant during all future periods from the end of current eradication to the next time when the disease is detected, and zero at other times. Incursion costs, $c = c_e + c_i$, are positive and constant during all future periods from the time the disease is detected till the end of eradication, and zero at other times.
Probability of being eradicated

Incursion time ($t = k_1$)  $t = k_1 + T + w$  $t = k_1 + T + w + k_2$

Eradication time

Detection time

Figure 1. The recurring process of disease incursion and eradication

The expected discounted present value of the flow of costs of hazard reduction and eradication with a constant discount rate $r$, can be expressed as follows:

$$E(C) = q \left\{ \int_0^w pe^{-pk_1} \left( \int_0^{k_1+w} he^{-rt} dt + \int_{k_1+w}^{k_1+w+T} (c_e + c_i) e^{-rt} dt \right) \right. + \left. + q \left( \int_0^w pe^{-pk_1} \left[ \int_{k_1+w}^{k_1+w+T} he^{-rt} dt + \int_{k_1+w+2w+T}^{k_1+2w+2T} (c_e + c_i) e^{-rt} dt + L \right] \right) \right\} d k_1$$

$$+(1-q) \left\{ \int_0^w he^{-rt} dt + \int_{w}^{w+T} (c_e + c_i) e^{-rt} dt \right. \right. + \left. \left. \left. + q \left( \int_0^\infty pe^{-pj_1} \left[ \int_{w+T}^{w+j_1+T} he^{-rt} dt + \int_{2w+2T+j_1+T}^{2w+2T+j_1} (c_e + c_i) e^{-rt} dt + L \right] \right) \right] dj_1 \right\}.$$
As mentioned earlier, at the end of each eradication process, the process is in the same state as initially (that is, \( t = 0 \)), therefore,

\[
C_0 = \int_0^\infty p e^{-p t} \left[ \int_0^{k + w} h e^{-r t} dt + \int_{k + w}^{k + w + T} (c_e + c_i)e^{-r t} dt \right]
+ q \left( \int_0^\infty p e^{-p t} \left[ \int_0^{k + w} h e^{-r t} dt + \int_{k + w}^{k + w + T} (c_e + c_i)e^{-r t} dt + L \right] dk \right)
+ (1 - q) \left( \int_{k + w}^{k + w + T} h e^{-r t} dt + \int_{k + w + T}^{k + 2w + T} (c_e + c_i)e^{-r t} dt + L \right)
\]

\[
C_1 = \left[ \int_0^w h e^{-r t} dt + \int_w^{w + T} (c_e + c_i)e^{-r t} dt + q e^{-r(w + T)} C_0 + (1 - q)e^{-r(k + w + T)}C_1 \right]
\]

and

\[
E(C) = qC_0 + (1 - q)C_1 .
\]

The above two equations can be further simplified to:

\[
\frac{q p e^{-r(w + T)}}{r + p} C_0 + \frac{(1 - q)p e^{-r(w + T)}}{r + p} C_1 = \frac{h p e^{-r w} - (c_e + c_i) p e^{-r w} (1 - e^{-r T})}{r (p + r)} - \frac{h}{r},
\]

\[
q e^{-r(w + T)} C_0 + [(1 - q) e^{-r(w + T)} - 1] C_1 = \frac{h (e^{-r w} - 1)}{r} + \frac{(c_e + c_i)(e^{-r(w + T)} - e^{-r w})}{r}.
\]

Solving the above two equations jointly for \( C_0 \) and \( C_1 \), yields

\[
C_0 = \frac{p (c_e + c_i)e^{-r w}[1 - e^{-r T}] + h (r + p) - (1 - q) r (h e^{-r(w + T)} - p e^{-r w})}{r \left[ (r + p) - (r + p)(1 - q) e^{-r(w + T)} - q p e^{-r(w + T)} \right]},
\]
The expected discounted present value of the flow of costs of hazard reduction and incursion $E(C)$ can now be written as

$$E(C) = qC_0 + (1 - q)C_1$$

$$= \frac{(c_e + c_i) 1 - e^{-rT}}{r} \left[ e^{-rT} + \frac{1 - q}{p} e^{-rT} \right] + \frac{h}{r} \left[ \frac{1 - q}{p} e^{-rT} + \frac{1 - e^{-rT}}{r} \right].$$

Let $\alpha = \frac{1 - e^{-rT}}{r} \left[ e^{-rT} + \frac{1 - q}{p} e^{-rT} \right] + \frac{1}{r} \left[ \frac{1 - q}{p} e^{-rT} + \frac{1 - e^{-rT}}{r} \right]$, then the expected cost can be simply expressed as:

$$E(C) = \alpha \cdot \frac{c_e + c_i}{r} + (1 - \alpha) \cdot \frac{h}{r},$$

where $\alpha$ can be interpreted as the expected ‘discounted time’ from the time of detection to eradication as a proportion of the expected ‘discounted time’ for a whole episode. ‘Discounted time’ takes into account that — at positive discount rates — the economic value of a unit time is not the same over time.

So, equation (1) or (2) gives the expected discounted present value of the sum of the hazard reduction, eradication and incursion costs. From this equation it is a straightforward exercise to find the level of hazard reduction and eradication activities for which this expected value is smallest (see the example later).

The expected cost in special cases

**Special case 1: hazard reduction activity applies all the time**

The analysis above is based on the assumption that hazard reduction would not be carried out over the time from detection to eradication. If it is assumed that hazard
reduction applies at all times, then the expected discounted present value of hazard reduction, eradication and incursion costs is:

\[ E(C) = \alpha \cdot \frac{c_e + c_i}{r} + \frac{h}{r}. \]  

**Special case 2: no uncertainty on eradication**

(a) \( q = 1 \)

If there is no uncertainty on eradication, that means, it is known for sure that the disease is completely removed after the eradication process. In this case, the probability \( q \) becomes 1. Then,

\[ E(C) = \frac{(c_e + c_i) 1 - e^{-rw}}{r} e^{-rw} + \frac{h}{r} \left[ \frac{1}{p} + \frac{1 - e^{-rw}}{r} \right]. \]

(b) \( q = 0 \)

If it is known for sure that the disease can never be removed completely, then the probability \( q \) becomes 0. This leads to:

Equation (5) gives the formula derived in Cao and Klijn (2004), where it was assumed that there is no uncertainty about the success of eradication and that the disease is detected at the time of incursion.
\[ E(C) = \frac{(c_e + c_i) - e^{-rT} \left[ \frac{r}{p} e^{-rw} + \frac{1}{p} \right] + \frac{h}{r} \left[ \frac{1}{p} - \frac{1}{p} e^{-rw} + \frac{1}{r} \right]}{1 - e^{-rw} + \frac{1}{r}} \]

The last equality reflects how the process repeats itself over the infinite time horizon. In this case, the process is a deterministic process and does not depend on the hazard rate at all. Therefore, the hazard reduction activity is of no use in this case — that is, \( h \) should be zero. Eradication activity is still needed, and its optimal level can be obtained by minimising the expected cost above subject to \( T = T(c_e) \).

**Example**

A hypothetical example is used to illustrate how the optimal levels of hazard reduction and eradication can be obtained using the formulas derived above. The formulas require the values of parameters of the discount rate \( r \), the production and trade loss \( c_i \), the probability of being eradicated at the end of eradication process \( q \), the time to detection \( w \), the functional relationships between the hazard rate \( p \) and the hazard reduction activity \( h \), and between the duration of eradication process \( T \) and the eradication activity \( c_e \).

Let \( r = 0.05 \) per year, \( c_i = 1 \) million dollars per year, \( q = 0.9 \), \( w = 2 \) years, and the monotonically decreasing relationships between \( p \) and \( h \) given by \( p(h) = 0.4 e^{-0.08h} \), and between \( T \) and \( c_e \) given by \( T(c_e) = 0.02 + (1 - e^{-0.0002c_e^{0.8}}) \). The functions \( p(h) \) and \( T(c_e) \) are depicted in figure 2(a) and (b) respectively.

The results for the optimal combination of hazard reduction and eradication activities are shown in Table 1 for the cases of 90, 0 and 100 per cent probability of the disease being eradicated at the end of eradication process. Results are given for the case that \( h \) applies all the time as well as for the case that \( h \) applies only in periods that disease is not detected.

Optimal hazard reduction activity when \( h \) applies all the time is lower than that when \( h \) applies in disease ‘free’ periods only, while the change in optimal eradication activity is
opposite — that is, optimal eradication activity is higher when \( h \) applies all the time than that when \( h \) applies in disease ‘free’ periods only. This could be explained as follows: when \( h \) applies all the time, that means there is a higher cost (that is, \( h + c_e + c_l \)) incurred during the disease period than the cost (that is, \( c_e + c_l \)) in the same period when \( h \) applies only in the disease ‘free’ period. The higher cost during the disease period leads to the need to increase the eradication expenditure to shorten the disease period (that is, \( T \)) so that the expected cost can be minimised. This is the reason why the optimal eradication activity is higher when \( h \) applies all the time than that when \( h \) applies in disease ‘free’ periods only. To minimise the expected cost, higher eradication cost combines with lower hazard reduction cost.

When \( q = 0 \), as explained earlier, the system is deterministic and there is no need for hazard reduction. That is, the optimal \( h \) is zero. As it is certain that the disease will not be eradicated, this leads to lower eradication expenditure, compared with the case of uncertainty about success of the eradication.

Comparison of \( q = 100 \) per cent with \( q = 90 \) per cent is more difficult. Optimal hazard reduction activity \( h \) for \( q = 100 \) per cent is slightly lower than that for \( q = 90 \) per cent when hazard reduction activity applies in disease ‘free’ periods only. When \( q = 100 \) per cent, it is certain that the disease will be eradicated. This leads to higher eradication expenditure than when \( q = 90 \) per cent. However, the optimal hazard reduction cost can be higher or lower, depending on the increase in optimal eradication expenditure.

Sensitivities of the optimal combination of hazard reduction and eradication activities to the probability of the disease being eradicated (\( q \)), the time till detection (\( w \)) and the production and trade loss (\( c_l \)) are shown in figures 3, 4 and 5, respectively.

Explanation of the simultaneous effects on optimal hazard reduction and eradication activities is difficult, as there is interaction between these variables. For example, with a higher probability \( q \) of the disease being eradicated at the end of eradication process, a rational decision maker increases spending on eradication. This affects the hazard reduction activity, through the reduction in duration of the eradication process \( T \), which results in all future incursions being brought back in time. Hence, it increases the benefits from hazard reduction. Figure 3 shows that both eradication and hazard reduction expenditures move up as \( q \) increases, when \( q \) is small; when \( q \) becomes big (close to 1), the optimal hazard reduction activity moves down slowly as \( q \) increases, while optimal eradication activity continues moving up as \( q \) increases, all other parameters being unchanged. This movement of optimal hazard reduction and eradication activities can be analytically analysed by looking at the first order conditions of the optimisation and looking at how the sign changes when \( q \) changes. Also note that the optimal expected cost \( E(C) \) moves down as \( q \) increases.
Similarly, in figure 4, when the time till detection $w$ increases while all other parameters are kept constant, future incursions are shifted forward in time and the benefits of hazard reduction decrease. This leads to a rational decision maker decreasing spending on hazard reduction. The decrease in hazard reduction activity leads to a slightly increase in eradication activity (figure 4) to bring the expected cost to its minimum. Optimal expected costs decrease with increasing $w$, when all other parameters are kept unchanged; this is because a higher $w$ leads to a smaller probability of the system being in the detectable disease state which is associated with high incursion costs.

When production and trade losses increase, it is expected that a rational decision maker increases expenditures on both hazard reduction and eradication activities to minimise expected cost (see figure 5).

![Figure 2](image_url)

**Figure 2.** The functional relationships between the hazard rate $p$ and the level of the hazard reduction activity $h$; and between the duration of eradication process $T$ and the level of the eradication activity $c_e$. 
Table 1. Optimal combination of hazard reduction and eradication activities

<table>
<thead>
<tr>
<th></th>
<th>Optimal $h$ (S'000 / yr)*</th>
<th>Optimal $p$ (yearly) **</th>
<th>Optimal $c_e$ (S'000 / yr)*</th>
<th>Optimal $T$ (year) ***</th>
<th>Optimal $E(C)$ (S'000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q = 90$ per cent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$ applies in ‘free’ periods</td>
<td>68.25</td>
<td>0.207</td>
<td>483.54</td>
<td>1.97</td>
<td>7488</td>
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<td>$h$ applies all the time</td>
<td>47.34</td>
<td>0.231</td>
<td>511.41</td>
<td>1.92</td>
<td>7740</td>
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<tr>
<td>$q = 100$ per cent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$ applies in ‘free’ periods</td>
<td>67.19</td>
<td>0.208</td>
<td>500.64</td>
<td>1.94</td>
<td>7062</td>
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<tr>
<td>$h$ applies all the time</td>
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<td>0.229</td>
<td>530.46</td>
<td>1.89</td>
<td>7295</td>
</tr>
<tr>
<td>$q = 0$ per cent</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h$ applies in ‘free’ periods</td>
<td>0</td>
<td>0.40</td>
<td>240.89</td>
<td>2.72</td>
<td>13573</td>
</tr>
<tr>
<td>$h$ applies all the time</td>
<td>0</td>
<td>0.40</td>
<td>240.89</td>
<td>2.72</td>
<td>13573</td>
</tr>
</tbody>
</table>

* This compares with the production and trade loss $c_l = $1 million per year.
** This compares with $p = 0.4$ when $h = 0$ (that is, the unmitigated hazard scenario).
*** This eradication period compares with a period to detection of $w = 2$ years.
# $h$ applies in disease ‘free’ periods only.
Figure 3. Optimal hazard reduction activity $h$, optimal eradication activity $c_e$ and optimal expected cost $E(C)$ for alternative values of the probability $q$ of successful eradication at the end of eradication process, all other parameters being constant.

Figure 4. Optimal hazard reduction activity $h$, optimal eradication activity $c_e$ and optimal expected cost $E(C)$ for alternative values of the time till detection $w$, all other parameters being constant.
Figure 5. Optimal hazard reduction activity $h$, optimal eradication activity $c_e$ and optimal expected cost $E(C)$ for alternative values of the production and trade loss $c_i$, all other parameters being constant.

Discussion
To determine the optimal combination of hazard reduction and eradication activities, it is required to know the production and trade loss $c_i$ during the incursion period; the probability $q$ of the disease being eradicated at the end of eradication process; the time till detection of the disease $w$; the discount rate $r$; the functional relationship $p(h)$ between the measure of hazard reduction $h$ and the hazard rate $p$; and the functional relationship $T(c_e)$ between the measure of eradication activity $c_e$ and the duration of eradication process $T$.

As Cao and Klijn (2004) discussed, the production and trade loss $c_i$ does not have to be constant over the periods from detection to the end of eradication process. In other words, $c_i$ can be a function of time $t$ — that is, $c_i=c_i(t)$. For example, $c_i(t)$ can be monotonically increasing, and can be linked to the biological dynamics of disease spread (see Kompas et al. 2003). For the formulae presented in this paper to continue to hold, the function $c_i(t)$ only needs to be the same in all incursion episodes.

The analytical expression for the discounted expected cost derived in this paper relies on two features: (1) every eradication process applied from detection to the end of the eradication process is identical and (2) that the hazard reduction cost flow is constant over time, such that the hazard rate and the expected incursion time from the last
eradication are identical. These features also require other parameter values to remain constant over time. It may be possible to widen the requirements that hazard reduction costs, hazard rate, expected incursion time and eradication costs are identical for all episodes to them having some functional relationships from episode to episode (for example, the values of these variables in the next episode are functions of their values in the current episode). This requires further investigation before any conclusions can be made. In some systems the features required above may not be present. For example, the hazard rate may change over time independently of hazard reduction activity or the probability of the disease being eradicated may change autonomously. In such cases, numerical simulations of these systems can be done using Monte-Carlo methods.

Although the assumptions used to derive the analytical expression of the expected cost in this paper are unlikely to be fully realistic, the analytical framework presented still captures essential elements of the decision making process for disease incursions and offers additional insights into the problem due to its analytical tractability. For example, the analytical expression for expected discounted future costs can be used to show how changes in parameter values affect decisions by looking at the partial derivatives of expected cost with respect to these parameters. Numerical simulations may only be able to provide such information at higher cost due to the complexity of the problem and the extensive computations involved.

The ability to provide quick solutions to disease incursion problems is another advantage of the framework presented in this paper. Solutions of the framework can be achieved within one or two seconds using a Pentium IV CPU computer for a given set of parameter values. This gives a huge advantage to applications of the framework by generating decisions for real-time management of disease incursions under various scenarios or assumptions within minutes. Imagine what the cost could be per minute without a proper solution if a disease incursion has already happened — could a decision maker afford a slow and delayed solution?
References


