Appendices to Working Paper 18/00

Appendix A: Equations in the TRYM model

In the sections below, references to *Documentation* are to the “Documentation of the Treasury Macroeconomic (TRYM) Model of the Australian Economy” (Commonwealth Treasury, 1996).

**A.1 Wage-setting equation in the TRYM model**

In the text (on page 23) the TRYM model wage equation is written as

\[\pi_w - \pi_{c_t} - \lambda = -a_2 \times \text{Union}_t \times \left[ c_1 \times \Delta UR_{t-1}^+ + (1-c_1) \times \Delta UR_{t-1}^- \right] - a_3 \times \Delta^4 \text{Central}_t \]

\[+ a_4 \times \frac{N \text{AIRU}_{t-1} - UR_{t-1}}{UR_{t-1}} + a_5 \times Q743_t.\]

Taking the expected-inflation and labour-augmenting technological progress terms (\(\pi_{c_t}^e\) and \(\lambda\)) to the right-hand side of the equation and substituting for expected inflation gives:

\[\pi_w = \lambda + \frac{\pi_{c_{t-1}}}{1 + a_1 + a_2^2 + a_3^3} + a_1 \pi_{c_{t-2}} + a_1^2 \pi_{c_{t-3}} + a_1^3 \pi_{c_{t-4}} + a_2 \times \text{Union}_t \times \left[ c_1 \times \Delta UR_{t-1}^+ + (1-c_1) \times \Delta UR_{t-1}^- \right] - a_3 \times \Delta^4 \text{Central}_t \]

\[+ a_4 \times \frac{N \text{AIRU}_{t-1} - UR_{t-1}}{UR_{t-1}} + a_5 \times Q743_t.\]

This is shown in the TRYM model documentation as:

\[\Delta \ln \left\{ \frac{RWT_t}{NH_t} \right\} = \frac{2}{4} + \Delta \ln \frac{PCON_{t-1}}{PCON_{t-1}} + a_1 \times \Delta \ln PCON_{t-1} + a_1^2 \times \Delta \ln PCON_{t-3} + a_1^3 \times \Delta \ln PCON_{t-4} + a_2 \times \Delta \ln RUM_{t-1} \times \left[ c_1 \times \Delta \text{RNUN}_{t-1} + (1-c_1) \times \Delta \text{RNUN}_{t-1} \right] - a_3 \times QCC_{t-1} - QCC_{t-4} \]

\[+ a_4 \times \frac{N \text{AIRU}_{t-1} - RNU_{t-1}}{RNU_{t-1}} + a_5 \times Q743_t.\]

In this equation, \(RWT_t\) is average earnings in quarter \(t\), on a national accounts basis, and \(NH_t\) is the number of hours worked per employee. Thus \(\Delta \ln (RWT_t/NH_t)\) is the change in the logarithm of nominal earnings per hour—the rate of nominal wage inflation. \(\lambda\) is the (estimated) average annual rate of labour-augmenting technical progress (in the text \(\lambda\) represents the quarterly rate of technological progress). \(PCON_t\) is the price deflator of private consumption in quarter \(t\), so that \(\Delta \ln (PCON_{t-1})\) is consumer price inflation on quarter \(t-i\). \(RUM_t\) is union membership as a proportion of total employees and \(QCC_t\) is a dummy variable for centralised wage-fixing systems. \(RNU_t\) is the unemployment rate in percentage points and \(\Delta RNU_t\) is the change in the unemployment rate between quarter
\( t - 1 \) and quarter \( t \). \( \Delta RNUP_t \) is the positive part of the change in the unemployment rate—it equals \( \Delta RNU_t \) when \( \Delta RNU_t \) is positive and equals zero when \( \Delta RNU_t \) is negative. Similarly, \( \Delta RNUN_t \) is the negative part of the change in the unemployment rate—it equals \( \Delta RNU_t \) when \( \Delta RNU_t \) is negative and equals zero when \( \Delta RNU_t \) is positive. \( Q743_t \) is a dummy variable to capture the effect on wages of the metal-trades wage decision in 1974(3); it takes the value of one in 1974(3) and zero in all other periods.

\( NAIRU_t \) is the value of the NAIRU in period \( t \), expressed in percentage points. In the text (on page 19) it is given by the formula,

\[
NAIRU_t = \begin{cases} 
UR_t^{ADJ} + WSo & \text{until 1973(4)}, \\
UR_t^{ADJ} + WS & \text{from 1974(1)}. 
\end{cases}
\]

The equivalent expression in the TRYM documentation is:

\[
NAIRU_t = (RNUST_t + WS) \times (1 - Q741_t) + (RNUST_t + WSo) \times Q741_t,
\]

where \( RNUST_t \) is the unemployment rate adjusted for search effectiveness. \( Q741_t \) is a dummy variable to capture a trend break in the NAIRU in the first quarter of 1974; it takes the value one up to 1973(4) and the value zero from 1974(1) on. \( WS \) and \( WSo \) are parameters to be estimated; they are designed to capture the effects of search effectiveness on the NAIRU for the periods from 74(1) and up to 73(4), respectively.

The estimated (short-run) wage equation for the period 1971(1) to 1999(2) is:

\[
\Delta \ln \left( \frac{RWT}{NH} \right) = \frac{\lambda}{4} + 0.340 \times \Delta \ln PCON_{t-1} + 0.271 \times \Delta \ln PCON_{t-2} + 0.216 \times \Delta \ln PCON_{t-3} + 0.173 \times \Delta \ln PCON_{t-4} - 0.0004 \times RUM_t \times \left[ \frac{0.367 \times \Delta RNUP_{t-1} + 0.633 \times \Delta RNUN_{t-1}}{1.51} \right] - 0.018 \times (QCC_t - QCC_{t-1}) - 0.011 \times \frac{RNU_{t-1} - NAIRU_{t-1}}{RNU_{t-1}} \bigg|_{(2.95)} + 0.075 \times Q743_t. \bigg|_{(5.42)}
\]

The numbers in parentheses below the parameter estimates are \( t \)-ratios. The estimated value of the coefficient \( a_1 \) in the expression for expected inflation is 0.798; its \( t \)-ratio is 2.99. The estimated equation for the NAIRU is:

\[
NAIRU_t = \begin{cases} 
RNUST_t + 3.70 & \text{until } 1973(4), \\
3RNUST_t + 1.83 & \text{from 1974(1)}. 
\end{cases}
\]

According to Downes and Bernie (1999) this implies a NAIRU of 4.05 per cent for 1971 to 1973 and of 6.45 per cent for 1974 on.
A.2 The Beveridge curve in the TRYM model

In the text (on page 31), the Beveridge curve is written as,

\[
\Delta \ln(UR_t) = a_1 \times \Delta \ln(VR_t) + a_2 \times \Delta LGF_t - a_0 \times \left[ \ln(UR_{t-1}) - \left[ c_0 + LGF_{t-1} + c_1 \times \ln(VR_{t-1}) \right] \right],
\]

where the logistical growth function is

\[
LGF_t = \frac{c_2}{1 + \exp \left( \frac{c_3 - t}{c_4} \right)}, \quad c_4 > 0.
\]

In the TRYM model documentation the short-run Beveridge curve is shown as:

\[
\Delta \ln(RNU_t) = a_1 \times \Delta \ln \left( \frac{NVA_t}{NLF_t} \right) + a_2 \times \Delta LGF_t - a_0 \times \left[ \ln(RNU_{t-1}) - \left[ c_0 + LGF_{t-1} + c_1 \times \ln \left( \frac{NVA_{t-1}}{NLF_{t-1}} \right) \right] \right],
\]

where \( RNU_t \) is the unemployment rate in quarter \( t \), \( NVA_t \) is the number of unfilled vacancies and \( NLF_t \) is the civilian labour force (so \( NVA_t / NLF_t \) is the vacancy rate). The logistical growth function is:

\[
LGF_t = \frac{c_2}{1 + \exp \left( \frac{c_3 - QTIME_t}{c_4} \right)},
\]

where \( QTIME_t \) is a linear time trend measured in years with value 0 at the end of 1997/start of 1998. The model is quarterly with observations taken as referring to the middle of the quarter, so \( QTIME_t \) equals –0.125 for the December quarter of 1997 and +0.125 for the March quarter of 1998.

The estimated (short-run) Beveridge curve for the period 1967(3) to 1999(2) is:

\[
\Delta \ln(RNU_t) = -0.206 \times \Delta \ln \left( \frac{NVA_t}{NLF_t} \right) + 2.271 \times \Delta LGF_t - 0.157 \times \left[ \ln(RNU_{t-1}) - \left[ 1.304 + LGF_{t-1} - 0.773 \times \ln \left( \frac{NVA_{t-1}}{NLF_{t-1}} \right) \right] \right],
\]

1 Downes and Bernie (1999) show a plus sign before the error-correction coefficient, \( a_0 \), but give an estimated value that is positive. This would imply that a “high” value of the unemployment rate causes the unemployment rate to increase further. They also show a minus sign before the coefficient, \( c_1 \), but give an estimated value that is negative. This would imply a positive relationship between the unemployment rate and the unemployment rate and the vacancy rate—an upward-sloping Beveridge curve. In both cases the sign before the coefficient is treated here as the typo.
Note that the scaling of the coefficients assumes that both the unemployment rate (RNUt) and the vacancy rate (NVAt/NLFt) are expressed in percentage points. The estimated logistical function is:

\[ LGF_t = 0.419 \left( \frac{1 + \exp \left[ \frac{-23.276 - QTIME_t}{415.49} \right]}{0.198} \right) \cdot \]

### A.3 Labour supply in the TRYM model

The long-run labour supply equation is shown in the text (on page 33) as

\[ \ln(PR_t) = c_7 \times Female_t \times \ln(ER_{t,POP}) + Trend_t + \ln(Dem_t), \]

or using the definitions of the participation rate and the employment rate as fractions of the working age population:

\[ \ln \left( \frac{LF_t}{POP_{t,15-64}} \right) = c_7 \times Female_t \times \ln \left( \frac{E_t}{POP_{t,15-64}} \right) + Trend_t + \ln(Dem_t). \]

If we multiply both sides of this expression by long-run average hours per employee we get:

\[ \ln \left( \frac{LF_t \times H_{t,LR}}{POP_{t,15-64}} \right) = Trend_t + \ln(Dem_t) + \ln(H_{t,LR}) + c_7 \times Female_t \times \ln \left( \frac{E_t}{POP_{t,15-64}} \right). \]

There are three different versions of this formula in Downes and Bernie (1999) and Documentation. The long-run labour-supply relationship in the error-correction term in the short-run labour supply equation shown in the Appendix C to Documentation (page 10.14) is the same as the one used here. It is shown as:

\[ \ln \left( \frac{NLF_t \times NHLR_t}{NPADA_t} \right) \]

\[ = Trend_t + \ln(QDEML_t) + \ln(NHLR_t) + c_7 \times QNLF_t \times \ln \left( \frac{NET_t}{NPADA_t} \right), \]

where NLFt is the civilian labour force, NHLRt is the desired level of hours worked (which is exogenous), NPADA, is the adult population aged between 15 and 64 (exogenous), QDEMLt is an index reflecting changes in the average participation rate due to demographic changes, NETt is total civilian employment, and QNFLFt is a dummy variable for female participation. TRENDt is a variable time trend designed to capture the upward trend in the participation rate due to factors other than changes in the age composition of the working-age population. It has the same form as Trendt in the text.

The long-run labour-supply relationship in the error-correction term in the short-run labour supply equation in the main body of Downes and Bernie (1999) and Documentation (Downes and Bernie, page 41, and Documentation, page 5.3) differs

---

\(^2\) There are some differences between the trend terms as shown in Appendix C in Documentation and as shown in the main texts of Documentation and Downes and Bernie (1999). The form in the text here and this appendix is the one in Appendix C in Documentation.
from the one here by not multiplying the employment term by the female-participation dummy. It is written as:

$$\ln\left( \frac{NLF_t \times NHLR_t}{NPADA_t} \right) = TRENDS_t + \ln(QDEML_t) + \ln(NHLR_t) + c_7 \times \ln\left( \frac{NET_t}{NPADA_t} \right).$$

Finally, in the explicit discussion of the long run in Downes and Bernie (page 40) and Documentation (page 5.2), the long-run labour supply equation is shown as:

$$\ln\left( \frac{NLF_t \times NHLR_t}{NPADA_t} \right) = TRENDS_t + \ln(QDEML_t) + c_7 \times \ln\left( \frac{NET_t \times NHLR_t}{NPADA_t} \right).$$

In the expression, but not in the previous one, the $\ln(NHLR_t)$ term on the right-hand side is multiplied by the coefficient $c_7$.

The short-run labour supply equation is written in the text (on page 35) as:

$$\ln(PR_{t+1}) = a_0 \times \left[ \ln(PR_t) + a_1 \times Female_t \times \left[ \Delta \ln\left( \frac{LD_{t}^{PRIV}}{POP_{t}^{15-64}} \right) - a_6^{id} \times Priv_t - \frac{\mu_t}{4} \right] + a_2 \times Female_t \times \left[ \Delta \ln\left( \frac{E_{t}^{GOV} + E_{t}^{PUB}}{POP_{t}^{15-64}} \right) + a_6^{id} \times Priv_t \right] + \Delta Trend_t + \Delta \ln(Dem_t) \right] + (1 - a_0) \times \left[ Trend_t + \ln(Dem_t) + c_7 \times Female_t \times \left[ \ln(E_{t}^{RO}) - \frac{\mu_t}{5} \right] \right].$$

If we add the log of long-run average hours per employee in period $t$, $\ln(H_{t}^{LR})$ to both sides, add and subtract the product of $a_0$ and the log of lagged average hours from the right-hand side, and or use the definitions of the participation rate and the employment rate as fractions or the working age population, we get:

---

3 In both Downes and Bernie and the main body of the text of Documentation, the coefficient $c_0$ is shown both as part of the $TREND$ variable and separately in the equation for labour supply. It is assumed here that it should appear only as part of $TREND$.  

---
\[
\ln \left( \frac{LF_t \times H_{15-64}^LR}{POP_{15-64}} \right) \\
= a_0 \times \left\{ \ln \left( \frac{LF_{t-1} \times H_{15-64}^{LR}}{POP_{15-64}} \right) + \Delta \ln \left( \frac{LD_{t}^{PRIV}}{POP_{15-64}} \right) - a_6^{ld} \times Priv_i - \frac{u_i^nebd}{4} \right\} \\
+ a_2 \times Female_i \times \left\{ \Delta \ln \left( \frac{E_{t}^{GOV} + E_{t}^{PUB}}{POP_{15-64}} \right) + a_6^{ld} \times Priv_i \right\} \\
+ \Delta Trend_i + \Delta \ln(Dem_i) + \Delta \ln(H_{15-64}^{LR}) \\
+ (1 - a_0) \times \left\{ \text{Trend}_i + \ln(Dem_i) + \ln(H_{15-64}^{LR}) + c_7 \times Female_i \times \left[ \ln \left( \frac{E_{t}}{POP_{15-64}} \right) - \frac{u_i^nebd}{5} \right] \right\}.
\]

In the TRYM model documentation this is shown as:

\[
\ln \left( \frac{NLF_t \times NHLR_t}{NPADA_t} \right) \\
= a_0 \times \left\{ \ln \left( \frac{NLF_{t-1} \times NHLR_{t-1}}{NPADA_{t-1}} \right) + \Delta \ln \left( \frac{NEBD_{t}}{NPADA_{t}} \right) - a_6^{ld} \times PRIV_t - \frac{u_i^nebd}{4} \right\} \\
+ a_2 \times QNLF_t \times \left\{ \Delta \ln \left( \frac{NEGG_t + NEGE_t}{NPADA_t} \right) + a_6^{ld} \times PRIV_t \right\} \\
+ \Delta Trend_t + \Delta \ln(QDEML_t) + \Delta \ln(NHLR_t) \\
+ (1 - a_0) \times \left\{ \text{Trend}_t + \ln(QDEML_t) + \ln(NHLR_t) \right\} \\
+ c_7 \times QNLF_t \times \left[ \ln \left( \frac{NET_t}{NPADA_t} \right) - \frac{u_i^nebd}{5} \right],
\]

where \(NLF_t\) is the civilian labour force, \(NHLR_t\) is the desired level of hours worked (which is exogenous), \(NPADA_t\) is the adult population between 15 and 64 (exogenous), \(NEBD_t\) is effective labour demand (employees plus vacancies) of the private business sector, \(NEGG_t\) is general government employment (exogenous), \(NEGE_t\) is public enterprise employment and \(NET_t\) is total civilian employment. \(QNLF_t\) is a dummy variable for the proportion of females in the labour force, \(QPRIV_t\) represents the effects of privatisation, \(QDEML_t\) is an index reflecting changes in the average participation rate due to demographic changes, \(a_6^{ld}\) is a parameter from the labour demand equation for

---

4 The variable \(QNLF_t\) appears in the long-run encouraged-worker effect term in Appendix C of Documentation, but not in Downes and Bernie (1999) or the main text of Documentation.
the private business sector, and $u_{t}^{nebd}$ is the residual from that equation. The trend is given by:

$$TREND_t = c_0 + \frac{c_1}{1 + \exp(c_2 \times [QTIME_t + c_3])} + c_4 \times \left( QTIME_t - \text{pos}[QTIME_t + c_5] \right) + c_6 \times \left[ 4 \times \left( 1 + \exp\left( -\frac{\text{pos}[QTIME_t + c_5]}{c_6} \right) \right)^{-2} \right],$$

where $\text{pos}[x]$ equals $x$ if $x > 0$ and equals zero otherwise. $QTIME$ is a time trend measured in years.

The estimated short-run labour-supply equation for the period 1971(2) to 1999(2) is:

$$\ln\left( \frac{NLF_t \times NHLR_t}{NPADA_t} \right)$$

$$= 0.611 \times \ln\left( \frac{NLF_{t-1} \times NHLR_{t-1}}{NPADA_{t-1}} \right) + \Delta TREND_t + \Delta \ln(QDEML_t) + \Delta \ln(NHLR_t)$$

$$+ 0.341 \times QNLF_t \times \left( \Delta \ln\left( \frac{NEBD_t}{NPADA_t} \right) - 0.000001 \times QPRIV_t - \frac{u_{t}^{nebd}}{4} \right)$$

$$+ 0.145 \times QNLF_t \times \left[ \Delta \ln\left( \frac{NEGD_t + NEGE_t}{NPADA_t} \right) + 0.000001 \times QPRIV_t \right]$$

$$+ 0.389 \times \left( TREND_t + \ln(QDEML_t) + \ln(NHLR_t) \right)$$

$$+ 0.400 \times QNLF_t \times \left[ \ln\left( \frac{NET_t}{NPADA_t} \right) - \frac{u_{t}^{nebd}}{5} \right].$$

The estimated trend is:
Note that the value of \( c_6 \) (that is, 5.0) is imposed rather than estimated.

**A.4 Average hours worked in the TRYM model**

The long-run hours-worked equation is shown in the text (on page 37) as
\[
TREND_{t} = 1.01 + \frac{-0.99}{(4.49)} \left[ 1 + \exp \left( -0.15 \times \left[ QTIME_{t} + 18.42 \right] \right) \right] \\
+ \frac{0.04 \times 5.36}{(32.10)} \left[ QTIME_{t} - \text{pos} \left[ QTIME_{t} + 14.47 \right] \right] \left[ 1 + \exp \left( \frac{-4}{\text{pos} \left[ QTIME_{t} + 14.47 \right]} \right) - 2 \right].
\]

The short-run hours-worked equation is written in the text (on page 38) as:
\[
\ln(H_t) = a_0 \times \left[ \ln(H_{t-1}) + a_1 \times \left[ \Delta \ln(Y_{PRIV}) - \text{Growth}_{t} \right] + a_2 \times \left[ \Delta \ln(Y_{PRIV}) - \text{Growth}_{t-1} \right] \right] \\
+ a_3 \times \left[ \ln \left( \frac{I_{DWell}}{I_{1-10}} \right) - \frac{\text{Growth}_t - \text{Growth}_{t-10}}{2} \right] \\
+ a_4 \times \left[ \ln(VR_t) - \ln(VR_{EXP}) \right] + \Delta H_t^{LR} \\
+ (1 - a_1) \times H_t^{LR}
\]

In the TRYM model documentation this is shown as:
\[
\ln(NH_t) = a_0 \times \left\{ \ln(NH_{t-1}) + a_1 \times [\Delta \ln(GBA_t) - GR_t] + a_2 \times [\Delta \ln(GBA_{t-1}) - GR_{t-1}] \right. \\
+ a_3 \times \left[ \frac{\ln(IDW_t / IDW_{t-10})}{10} - \frac{GR_t - GR_{t-10}}{2} \right] \\
+ a_4 \times \left[ \ln \left( 100 \times \frac{NVA_t}{NLF_t} \right) - \frac{\ln(RNU_t - u^{NVA_t}_{t} - LRNUB_t)}{c^1_{BC}} \right] + \Delta NHLR_t \left\} \\
+ (1 - a_0) \times NHLR_t,
\]

where \( NH_t \) is the number of hours worked per employee in quarter \( t \), \( GBA_t \) is the rain-adjusted output of the private business sector, \( GR_t \) is the steady-state growth rate of the economy, \( IDW_t \) is dwelling investment, \( NVA_t \) is the number of unfilled vacancies, \( NLF_t \) is the civilian labour force, \( RNU_t \) is the unemployment rate, \( LRNUB_t \) is the trend term in the long-run Beveridge curve equation, \( u^{NVA_t}_{t} \) is the residual from the Beveridge curve equation, and \( c^1_{BC} \) is a coefficient from the Beveridge curve equation.

The estimated expression for desired long-run average hours for the period 1971(2) to 1999(2) is:

\[
LGF_t = 0.134 + \frac{-0.133}{(3.09)} \times \left[ 1 + \exp \left( -0.387 \times \left( QTIME_t + 24.51 \right) \right) \right],
\]

and the estimated short-run hours-worked equation for the period 1971(2) to 1999(2) is:

\[
\ln(NH_t) = 0.474 \times \left\{ \ln(NH_{t-1}) + 0.076 \times [\Delta \ln(GBA_t) - GR_t] + 0.163 \times [\Delta \ln(GBA_{t-1}) - GR_{t-1}] \right. \\
+ 0.112 \times \left[ \frac{\ln(IDW_t / IDW_{t-10})}{10} - \frac{GR_t - GR_{t-10}}{2} \right] \\
+ 0.010 \times \left[ \ln \left( 100 \times \frac{NVA_t}{NLF_t} \right) - \frac{\ln(RNU_t - u^{NVA_t}_{t} - LRNUB_t)}{-0.773} \right] + \Delta LGF_t \left\} \\
+ 0.526 \times LGF_t.
\]

**A.5 Labour demand in the TRYM model**

In the text (on page 39) the equation for long-run labour demand by the government enterprise sector is shown as:

\[
\ln(E^{GE}_{t} \times H_{t}) = \ln(K^{GE}_{t-1}) + c_0 - (A_L + c_1) t.
\]
In Documentation it is written as:

$$\ln(NEGE_t \times NH_t) = \ln(KGE_{t-1}) + c_0 - c_1 \times QTIME_t - \lambda \times QTIME_t,$$

where \(NEGE_t\) is public enterprise employment, \(NH_t\) is the number of hours worked per employee in the economy, \(KGE_t\) is the government enterprise capital stock, \(QTIME_t\) is a time trend measured in years, and \(\lambda\) is the estimated rate of Harrod-neutral technological progress in the private business sector (at an annual rate).

In the text (on page 41), the equation for short-run labour demand in the government enterprises sector is shown as:

$$\Delta \ln \left( \frac{E_{t-1}^{GE} \times H_{t-1}}{K_{t-2}^{GE}} \right) = -\lambda_{GE} + a_1 \times \Delta \ln \left( \frac{E_{t-1}^{GE} \times H_{t-1}}{K_{t-2}^{GE}} \right) + \lambda_{GE},$$

$$a_0 \times \left[ c_0 - \lambda_{GE} t - \ln \left( \frac{E_{t-1}^{GE} \times H_{t-1}}{K_{t-2}^{GE}} \right) \right].$$

In Documentation it is written as:

$$\Delta \ln \left( \frac{NEGE_t \times NH_{t-1}}{KGE_{t-1}} \right) = -\frac{c_1 + \lambda}{4} + a_1 \times \Delta \ln \left( \frac{NEGE_{t-1} \times NH_{t-1}}{KGE_{t-2}} \right) + \frac{c_1 + \lambda}{4},$$

$$+ a_0 \times \left[ c_0 - (c_1 + \lambda) \times QTIME_t - \ln \left( \frac{NEGE_{t-1} \times NH_{t-1}}{KGE_{t-2}} \right) \right].$$

where \(NEGE_t\) is public enterprise employment, \(NH_t\) is the number of hours worked per employee in the economy, \(KGE_t\) is the government enterprise capital stock, \(QTIME_t\) is a time trend measured in years, and \(\lambda\) is the estimated rate of Harrod-neutral technological progress in the private business sector (at an annual rate).

The estimated short-run labour-demand equation for the government enterprise sector for the sample period 1970(1) to 1995(3) is:

$$\Delta \ln \left( \frac{NEGE_t \times NH_{t-1}}{KGE_{t-1}} \right)$$

$$= - \frac{0.036 + 0.008}{4} + 0.108 \times \left( \Delta \ln \left( \frac{NEGE_{t-1} \times NH_{t-1}}{KGE_{t-2}} \right) + \frac{0.036 + 0.008}{4} \right)$$

$$+ 0.067 \times \left( \frac{0.036 + 0.008}{1.74} \times QTIME_t - \ln \left( \frac{NEGE_{t-1} \times NH_{t-1}}{KGE_{t-2}} \right) \right).$$

In the text (on page 40) the equation for long-run labour demand by the private business sector is shown as:

$$\ln \left( E_t^{BUS} + V_t \right) = \ln \left( Y_t^{BUS} \right) - \ln \left( H_t \right) - \lambda t + \sigma \times \ln(\alpha) - \sigma \times \ln \left( \frac{W_t^{BUS}}{P_t^{BUS}} \right) - \lambda t.$$
In Downes and Bernie (1999) and Documentation it is written as:

\[
\ln(NEBD_t) = \ln(GBA_t) - \ln(NH_t) - \lambda_t \times QTIME_t + \sigma \times \ln(\alpha)
\]

\[
- \sigma \times \left[ \ln \left( \frac{RWH_t \times RTPRB_t}{PGB_t} \right) - \lambda_t \times QTIME_t \right],
\]

where \(NEBD_t\) is effective labour demand by the private business sector, \(GBA_t\) is rain adjusted private output, \(NH_t\) is the number of hours worked per employee in the economy, \(RWH_t\) is the hourly business wage, \(RTPRB_t\) represents taxes on payrolls and fringe benefits, \(PGB_t\) is the price of domestic output, \(QTIME_t\) is a time trend measured in years, \(\sigma\) is the elasticity of substitution between capital and labour, \(\alpha\) is the CES parameter on labour, and \(\lambda_t\) is the estimated rate of Harrod-neutral technological progress in the private business sector (at an annual rate).

In the text (on page 42), the equation for short-run labour demand in the government enterprises sector is shown as:

\[
\Delta \ln(E_{t}^{BUS} + V_{t}) = n_t^8 - a_i \times \sigma \times \left[ \Delta \ln \left( \frac{E_{t}^{BUS}}{P_{t}^{BUS}} \right) - \lambda_L \right] - (1 - a_z) \times \Delta \ln(H_t)
\]

\[
+ a_z \times \Delta \ln \left( \frac{E_{t-1}^{BUS}}{Y_{t-1}^{BUS}} \right) - n_{t-1}^4 - \lambda_{t-1} + a_z \times \Delta \ln \left( \frac{E_{t-1}^{BUS}}{Y_{t-1}^{BUS}} \right) - n_{t-1}^4 - \lambda_{t-1} + a_z \times \Priv_t
\]

\[
- a_y \times \left[ \Delta \ln \left( \frac{E_{t}^{BUS}}{P_{t}^{BUS}} \right) - \lambda_{t-1} \times (t-1) \right]
\]

In Downes and Bernie (1999) and Documentation it is written as:

\[
\Delta \ln(NEBD_t)
\]

\[
= \frac{\Delta^8 \ln(NPAD_t)}{8} - a_i \times \sigma \times \left[ \Delta \ln \left( \frac{RWH_t \times RTPRB_t}{PGB_t} \right) - \lambda_L \right] - (1 - a_z) \times \Delta \ln(NH_t)
\]

\[
+ a_z \times \Delta \ln(GBA_t) - \frac{\Delta^4 \ln(NPAD_t)}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-1}) - \frac{\Delta^4 \ln(NPAD_{t-1})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-2}) - \frac{\Delta^4 \ln(NPAD_{t-2})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-3}) - \frac{\Delta^4 \ln(NPAD_{t-3})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-4}) - \frac{\Delta^4 \ln(NPAD_{t-4})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-5}) - \frac{\Delta^4 \ln(NPAD_{t-5})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-6}) - \frac{\Delta^4 \ln(NPAD_{t-6})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-7}) - \frac{\Delta^4 \ln(NPAD_{t-7})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_z \times \Delta \ln(GBA_{t-8}) - \frac{\Delta^4 \ln(NPAD_{t-8})}{4} - \frac{\lambda_L}{4}
\]

\[
+ a_y \times \Delta \ln \left( \frac{RWH_{t-1} \times RTPRB_{t-1}}{PGB_{t-1}} \right) - \lambda_{t-1} \times QTIME_{t-1}
\]

\[
+ \sigma \times \left[ \Delta \ln \left( \frac{RWH_{t-1} \times RTPRB_{t-1}}{PGB_{t-1}} \right) - \lambda_{t-1} \times QTIME_{t-1} \right]
\]

where \(NEBD_t\) is effective labour demand by the private business sector, \(GBA_t\) is rain adjusted private output, \(NH_t\) is the number of hours worked per employee in the economy, \(RWH_t\) is the hourly business wage, \(RTPRB_t\) represents taxes on payrolls and fringe benefits, \(PGB_t\) is the price of domestic output, \(NPAD_t\) is the adult population, \(QPRIV_t\) is a privatisation dummy, \(QTIME_t\) is a time trend measured in years, \(\sigma\) is the
elasticity of substitution between capital and labour, $\alpha$ is the CES parameter on labour, and $\lambda$ is the estimated rate of Harrod-neutral technological progress in the private business sector (at an annual rate).

The estimated short-run labour demand equation for the private business sector for the sample period 1970(4) to 1999(2) is:

$$\Delta \ln(NEBD) = \frac{\Delta^t \ln(NPAD)}{8} - 0.150 \times 0.817 \left[ \Delta \ln \left( \frac{RWH \times RTPRB}{PGB} \right) - 0.012 \right]$$

$$- \left( 1 - 0.713 \right) \Delta \ln(NH) + 0.240 \left[ \Delta \ln(GBA) - \frac{\Delta^t \ln(NPAD)}{4} - 0.012 \right]$$

$$+ 0.106 \left[ \Delta \ln(GBA_{-1}) - \frac{\Delta^t \ln(NPAD_{-1})}{4} - 0.012 \right]$$

$$+ 0.085 \left[ \Delta \ln(GBA_{-2}) - \frac{\Delta^t \ln(NPAD_{-2})}{4} - 0.012 \right] + 0.000001 \times QPRIV,$$

$$\Delta \ln(PG) = 0.198 \left[ \Delta \ln \left( \frac{NEBD}{GBA_{-1}} \right) + \ln(NH_{-1}) + 0.012 \times \ln(QTIME_{-1}) - 0.817 \times \ln(QTIME_{-1}) \right],$$

$$\Delta \ln(PG) = 0.198 \left[ \Delta \ln \left( \frac{RWH \times RTPRB}{PGB_{-1}} \right) - 0.012 \times \ln(QTIME_{-1}) \right].$$

A.6 Relationship between the unemployment rate and the employment rate

If we write the long-run labour force as $LF$, the long-run population of working age as $POP$, the number of persons employed as $E$, then in the TRYM model, abstracting from changes in hours worked and trend changes in participation rates,

$$\ln \left( \frac{LF}{POP} \right) = c_7 \times \ln \left( \frac{E}{POP} \right),$$

where the coefficient $c_7$, the elasticity of the participation rate with respect to the employment rate, is estimated to be 0.400. It is convenient to denote the participation rate and the employment rate, as fractions of the working-age population, by $PR^{POP} \equiv LF / POP$ and $ER^{POP} \equiv E / POP$, so that $\ln(PR^{POP}) = c_7 \times \ln(ER^{POP})$.

Suppose we want to compare two scenarios for the economy, one with labour force and employment equal to $LF_0$ and $E_0$, and the other with labour force and employment equal to $LF_1$ and $E_1$, but with the same population in each case. If we use $\Delta$ to denote the difference in a variable between scenario 0 and scenario 1, then from the relationship between the participation rate and the employment rate:

$$\frac{\Delta PR^{POP}}{PR_0^{POP}} \approx c_7 \times \frac{\Delta ER^{POP}}{ER_0^{POP}} \quad \text{or} \quad \Delta PR^{POP} \approx \frac{LF_0}{E_0} \times c_7 \times \Delta ER^{POP} \equiv \frac{c_7}{ER_0^{LF}} \times \Delta ER^{POP},$$

where $ER^{LF}$ is the employment rate as a fraction of the labour force. That is, since $c_7$ equals 0.400, the proportionate change in the participation rate is approximately 40 per
cent as large as the proportionate change in the employment rate (where the participation and employment rates are measured as fractions of the working-age population). With a long-run unemployment rate (NAIRU) of seven per cent, the employment rate as a proportion of the labour force is 93 per cent, and a one percentage point change in the employment rate as a fraction of the working age population (or a one percentage point increase in employment given population) leads to an increase in the participation rate (or in the labour force, given population) of about 0.43 per cent.

Let us denote the unemployment rate relative to the labour force (the usual definition) by \( UR \equiv U / LF \), where \( U = LF - E \) is the number of persons unemployed. Then the difference in the unemployment rate between scenarios 0 and 1 is:

\[
\Delta UR = \frac{U_1}{LF_1} - \frac{U_0}{LF_0} = \frac{E_0 - E_1}{LF_0} = \frac{ER_{0,POP}}{PR_{0,POP}} - \frac{ER_{1,POP}}{PR_{1,POP}},
\]

since the population is the same under both scenarios. Rewriting the employment and participation rates under scenario 1 in terms of changes from scenario 0, and using the relationship between changes in the participation rate and changes in the employment rate, gives:

\[
\Delta UR = \frac{ER_{0,POP}}{PR_{0,POP}} - \frac{ER_{0,POP} + \Delta ER_{POP}}{PR_{0,POP} + \Delta PR_{POP}} \approx \frac{ER_{0,POP}}{PR_{0,POP}} - \frac{ER_{0,POP} + \Delta ER_{POP}}{PR_{0,POP} + c_7 \times \Delta ER_{POP} / ER_{0,POP}}.
\]

The final term on the right can be rewritten as:

\[
\frac{ER_{0,POP}}{PR_{0,POP}} \times \frac{1 + \Delta ER_{POP} / ER_{0,POP}}{1 + c_7 \times \Delta ER_{POP} / ER_{0,POP}}.
\]

Now, from the binomial theorem, this is approximately equal to:

\[
\frac{ER_{0,POP}}{PR_{0,POP}} \times \left[ 1 + (1 - c_7) \times \frac{\Delta ER_{POP}}{ER_{0,POP}} \right].
\]

Substituting this into the expression for the change in the unemployment rate gives:

\[
\Delta UR \approx -(1 - c_7) \times \frac{\Delta ER_{POP}}{PR_{0,POP}}.
\]

Since \( c_7 \) equals 0.4, a one percentage point increase in the employment rate as a fraction of the working age population, or in employment given population, leads to an 0.6 percentage point fall in the number of persons unemployed. If the participation rate as a fraction of the working-age population is 75 per cent, then a one percentage point increase in the number of persons employed leads to about an 0.8 percentage point fall in the unemployment rate.

**A.7 Profit maximisation by firms**

Consider a firm with a CES production of the form,

\[
Y_t = \left\{ \alpha \left[ \exp(\lambda_k t) L_t \right]^{\sigma - 1} + \beta \left[ \exp(\lambda_k t) K_t \right]^{\sigma - 1} \right\}^{\frac{\sigma}{\sigma - 1}},
\]

\[\text{where } \alpha, \beta, \lambda_k, \sigma, \sigma, \lambda_k \text{ are parameters.}\]
where $Y_t$ is the firm’s output in period $t$, $L_t$ is its labour input in hours, $K_t$ is its capital input, and $\sigma$ is the elasticity of substitution between labour and capital. There is both labour-augmenting and capital-augmenting technological progress—at rates $\lambda_L$ and $\lambda_K$, respectively.

For a given capital stock, output price, $P_t$, and nominal wage rate, $W_t$, the firm maximises its profits by choosing labour input to maximise $P_tY_t - W_tL_t$. It thus maximises it profits by choosing a level of labour input at which the marginal revenue product of labour, $P_t \times \partial Y_t / \partial L_t$, is equal to the nominal wage, or at which the marginal product of labour equals the real wage. For the CES production function, this implies:

$$\alpha \times \exp\left(\frac{\sigma-1}{\sigma} \times \lambda_L t\right) \times \left(\frac{Y_t}{L_t}\right)^{\frac{\frac{1}{\sigma}}{}} = \frac{W_t}{P_t}.$$  

This condition can be easier to manipulate if we take logs of both sides:

$$\ln(\alpha) + \frac{\sigma-1}{\sigma} \times \lambda_L t + \frac{1}{\sigma} \times \ln(Y_t) - \frac{1}{\sigma} \times \ln(L_t) = \ln\left(\frac{W_t}{P_t}\right).$$

We can manipulate this expression in a number of ways. First, we can solve for labour input to get labour demand as a function of output and the real wage:

$$\ln(L_t) = \sigma \times \ln(\alpha) + (\sigma-1) \times \lambda_L t + \ln(Y_t) - \sigma \times \ln\left(\frac{W_t}{P_t}\right).$$

Long-run demand for labour in the private business sector is based on this relationship. Second, we can substitute the expression for labour demand into the production function to get:

$$Y_t = \left\{ \alpha^\sigma \times \exp[(\sigma-1) \lambda_L t] \times \left(\frac{W_t}{P_t}\right)^{\frac{1}{\sigma}} \times \left[\frac{\exp(\lambda_K t) \times K_t}{Y_t}\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}.$$  

Solving this for output price gives:

$$P_t = \frac{W_t}{\alpha \times \exp(\lambda_L t)} \times \left\{ \frac{1}{\alpha} - \beta \times \left[\frac{\exp(\lambda_K t) \times K_t}{Y_t}\right]^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{1}{\sigma-1}}.$$  

This relationship is used as the basis of the long-run price setting equation for the private business sector.
Appendix B: Labour market equations in the AEM model

This appendix relates the labour market equations of the Access Economics Macro model as shown in the text above to their representation in the AEM model documentation, and provides some further details on the equations.

B.1 The wage-setting equation in the AEM model

In the text (on page 45), the wage-setting equation in the AEM model is shown as:

$$\frac{\Delta \ln(W_t)}{1+SG_i} - \pi_t^* - \lambda_t = a_0 + a_1 \times \frac{\Delta E_{t-1}}{LF_{t-1}} + a_2 \times \frac{1}{UR_{t-4}} + u_{t}^{WE} - u_{t-1}^{WE},$$

The wage-setting equation in the AEM model works in terms of the employment to labour force ratio, $E_t/LF_t = 1 - UR_t$, rather than the unemployment rate. Using this to substitute for the unemployment rate and rearranging terms gives:

$$\Delta \ln(W_t) = (1 + SG_i) \times \left( \lambda_t + a_0 + a_1 \times \frac{\Delta E_{t-1}}{LF_{t-1}} + a_2 \times \frac{1}{1 - E_{t-4}/LF_{t-4}} + \pi_t^* + u_{t}^{WE} - u_{t-1}^{WE} \right).$$

The corresponding expression in the AEM model documentation is:

$$\Delta \ln(W_{geAqet}) = \left[ \frac{ExoPdyLab_{t-1} + A00WgeAqe}{ExoPdyLab_{t-1}} \right] + \left[ \frac{ExoPdyLab_{t-1} + A00WgeAqe \times \DeltaLabEmpTot_{t-1}}{LabSup_{t-1}} \right] + \left[ \frac{A01WgeAqe \times 1}{1 - \text{LabEmpTot}_{t-4}/\text{LabSup}_{t-4}} \right],

[1 - A03WgeAqe - A04WgeAqe] \times \DeltaPcePci_{t-3} + A03WgeAqe \times \DeltaPcePci_{t-3} + A04WgeAqe \times PceEdi_{t-1} \times 4] + ResWgeAqe - ResWgeAqe_{t-1},$$

where $WgeAqet$ is average weekly earnings, $ExoWgeSgc_t$ is the wage adjustment for the superannuation guarantee, $ExoPdyLab_t$ is a labour efficiency index, $LabEmpTot_t$ is employment including defence personnel (in thousands), $LabSup_t$ is labour supply or the labour force including defence personnel (in thousands), $PcePci_t$ is the private consumption constant utility price index, $PceEdi_t$ is the expected domestic inflation rate, and $ResWgeAqe_t$ is the residual in the wage equation. The superannuation guarantee and productivity variables, $ExoWgeSgc$ and $ExoPdyLab$, are exogenous. Note that since the labour supply variable is defined to include employment in the defence forces, the unemployment rate shown in the text is the ratio of unemployment to the total labour force rather than the ratio to the civilian labour force. The term in braces on the right-hand side of the equation is expected inflation, $\pi_t^*$. It is a weighted average of past consumer price inflation ($\DeltaPcePci_{t-1}$ and $\DeltaPcePci_{t-3}$) and the lag of the expected domestic inflation rate $PcdEdi_t$. The expected domestic inflation rate depends on its past values, the RBA’s target inflation rate, and the previous year’s underlying inflation rate. It is given by:
\[ PceEdi_t = A00PceEdi \times PceEdi_{t-1} + A01PceEdi \times ExoPceRba_t, \]
\[ + \left(1 - A00PceEdi - A01PceEdi \right) \left( \frac{PceUnd_{t-1}}{PceUnd_{t-5}} - 1 \right) + ResPceEdi_t, \]
\[ + A00PceEdi \times ResPceEdi_{t-1}, \]

where \( ExoPceRba_t \) is the RBA target inflation rate (which is exogenous), \( PceUnd_t \) is the Treasury underlying price index, and \( ResPceEdi_t \) is the residual in the expected domestic inflation rate equation. The underlying inflation rate, in turn, depends on a weighted average of current and past values lags of the change in the private consumption deflator. It is given by the equation:

\[ PceUnd_t = \left(1 + ResPceUnd_t \right) \times PceUnd_{t-1} \]
\[ \times \left[ 0.5 \times \frac{PcePcd_t}{PcePcd_{t-1}} + 0.45 \times \frac{PcePcd_{t-1}}{PcePcd_{t-2}} + 0.05 \times \frac{PcePcd_{t-2}}{PcePcd_{t-3}} - 0.0004 \right], \]

where \( PcePcd_t \) is the private consumption deflator and \( ResPceUnd_t \) is a residual.

The estimated form of the wage-setting equation for the sample period 1976(1) to 1997(1), as reported in the Model Listing section of the documentation for the AEM model (pages 88–88) is:

\[ \Delta \ln(WgeAqe_t) = \left[ \frac{ExoPdyLab_t}{ExoPdyLab_{t-1}} - 0.0125 \right. \]
\[ + 1.2549 \times \frac{\Delta LabEmpTot_{t-1}}{LabSup_{t-1}} \]
\[ + \frac{0.0008 \times 1}{0.0004 \times 1 - LabEmpTot_{t-4}/LabSup_{t-4}} \]
\[ + \left(1 - 0.1519 - \text{??}\right) \times \Delta PcePci_{t-1} \]
\[ + 0.1519 \times \Delta PcePci_{t-3} \]
\[ \left. + \frac{? \times PceEdi_{t-1} - ResWgeAqe_t - ResWgeAqe_{t-1}}{4} \right] \]

where the numbers in square brackets below the coefficients are standard errors. The estimated value of the coefficient on the lagged domestic inflation rate was not reported in the documentation. In the discussion of the wage-setting equation on pages 30–31 of the documentation, however, the estimated value of \( a_0 \) is given as –0.02157 and the estimated value of \( a_1 \) as 0.00165 (no estimates are given for other coefficients). The estimated form of the equation for the domestic inflation rate for the sample period 1976(1) to 1997(1) is:

\[ PceEdi_t = 0.7 \times PceEdi_{t-1} + 0.25 \times ExoPceRba_t + 0.05 \times \left( \frac{PceUnd_{t-1}}{PceUnd_{t-5}} - 1 \right) \]
\[ + ResPceEdi_t + 0.7 \times ResPceEdi_{t-1}. \]
B.2 The labour supply equation in the AEM model

In the text (on page 47), the short-run unemployment equation in the AEM model is shown as:

\[ UR_t = a_0 \times UR_{t-1} + a_1 \times NAIRU + a_2 \times [\gamma(D_{i}^{LR}) - \lambda_i - \gamma(E_i)] + a_3 \times \Delta Replace_t \]

\[ + u_{t}^{LR} - a_0 \times u_{t-1}^{LR}, \]

The corresponding expression in the AEM model documentation is:

\[
\text{LabUneRte}_t = 100 \times \left[ \frac{A00\text{LabUne}_t \times \text{LabUneRte}_{t-1} - \frac{A01\text{LabUne}_t \times A02WgeAqe\_t}{A00WgeAqe}}{A00WgeAqe} \right]
\]

\[
+ \frac{A02\text{LabUne}_t}{A00\text{LabUne}_t} \left( \frac{\text{LabEmp}_t}{\text{LabEmp}_{t-1}} + \frac{\text{ExoPdyLab}_t}{\text{ExoPdyLab}_{t-1}} - \frac{\text{EcoDfdTrd}_t}{\text{EcoDfdTrd}_{t-1}} - 1 \right)
\]

\[
+ \frac{A03\text{LabUne}_t}{A00\text{LabUne}_t} \left( \frac{\text{ExoTsfUbeWge}_t - \text{ExoTsfUbeWge}_{t-1}}{\text{ExoTsfUbeWge}_{t-1}} \right)
\]

\[
+ \text{ResLabUneRte}_t - A00\text{LabUne}_t \times \text{ResLabUneRte}_{t-1},
\]

where \( \text{LabUneRte}_t \) is the unemployment rate (the ratio of the number unemployed to the civilian labour force), \( \text{LabEmp}_t \) is civilian employment (in thousands of persons), \( \text{ExoPdyLab}_t \) is a labour efficiency index, \( \text{EcoDfdTrd}_t \) is trend domestic final demand, \( \text{ExoTsfUbeWge}_t \) is the ratio of the unemployment benefit to the after-tax wage (the replacement ratio), and \( \text{ResLabUneRte}_t \) is the residual in the unemployment rate equation. The labour efficiency index and the replacement ratio are exogenous.

\( A00WgeAqe \) and \( A02WgeAqe \) are coefficients from the wage-setting equation and \( A02WgeAqe/A00WgeAqe \) is the negative of the NAIRU.

The values of the coefficients in the unemployment rate equation (other than those from the wage-setting equation) are imposed rather than estimated. The equation with the imposed values is:

\[
\text{LabUneRte}_t = 100 \times \left[ \frac{0.9 \times \text{LabUneRte}_{t-1} - 0.1 \times 0.0008}{-0.0125} \right]
\]

\[
- 0.3 \times \left( \frac{\text{LabEmp}_t}{\text{LabEmp}_{t-1}} + \frac{\text{ExoPdyLab}_t}{\text{ExoPdyLab}_{t-1}} - \frac{\text{EcoDfdTrd}_t}{\text{EcoDfdTrd}_{t-1}} - 1 \right)
\]

\[
+ 0.1 \times \left( \frac{\text{ExoTsfUbeWge}_t - \text{ExoTsfUbeWge}_{t-1}}{\text{ExoTsfUbeWge}_{t-1}} \right)
\]

\[
+ \text{ResLabUneRte}_t - 0.9 \times \text{ResLabUneRte}_{t-1}.
\]

B.3 Labour demand in the AEM model

In the text (on page 48), the private-sector labour-demand equation is shown in error-correction form as:
\[
\ln(E_t - E_t^{GOV}) - \ln[(1 + n_t) \times (E_{t-1} - E_{t-1}^{GOV})] \\
= a_0 - a_1 \times \ln\left(\frac{P_i^{EW} \times Micro_i}{P_t}\right) + a_3 \times \Delta YoverK_i + a_4 \times \Delta YoverK_{t-1} - a_5 \times t \\
- a_2 \times \ln\left[(1 + n_t) \times (E_{t-1} - E_{t-1}^{GOV} - E_{t-1}^{P.EQ})\right] + u^{LD}_t.
\]

This can be rearranged to be:

\[
\ln\left(\frac{E_t - E_t^{GOV}}{E_{t-1} - E_{t-1}^{GOV}}\right) = a_0 - a_1 \times \ln\left(\frac{P_i^{EW} \times Micro_i}{P_t}\right) + a_2 \times \ln\left[E_{t-1}^{P.EQ} \times (1 + n_t)\right] \\
+ a_3 \times \Delta YoverK_i + a_4 \times \Delta YoverK_{t-1} \\
+ (1 - a_2) \times \ln\left[(1 + n_t) \times (E_{t-1} - E_{t-1}^{GOV})\right] - a_5 \times t + u^{LD}_t.
\]

The corresponding expression in the AEM model documentation is:

\[
\ln(\text{LabEmpTot}_t - \text{ExoPubEgg}_t) \\
= A00\text{LabEmp} + A01\text{LabEmp} \times \ln\left(\frac{PcePrdEql_t \times ExoPdyMic_t}{PcePrd_t}\right) \\
+ A02\text{LabEmp} \times \ln[\text{LabEmpEq}_{t,-1} \times \exp(\text{PopGrw}_t)] \\
+ A03\text{LabEmp} \times \text{EcoProKap}_t + A04\text{LabEmp} \times \text{EcoProKap}_{t-1} \\
+ (1 - A02\text{LabEmp}) \times \ln[\exp(\text{PopGrw}_t) \times (\text{LabEmpTot}_{t-1} - \text{ExoPubEgg}_{t-1})] \\
+ A05\text{LabEmp} \times \text{ExoTimeFxd}_t + \text{ResLabEmpTot}_t.
\]

where LabEmpTot\(_t\) is total employment including defence personnel (in thousands), ExoPubEgg\(_t\) is general government employment (in thousands), PcePrdEql\(_t\) is the (short-run) equilibrium price index for domestic production, ExoPdyMic\(_t\) is a microeconomic reform index, PcePrd\(_t\) is the price index for domestic production, LabEmpEql\(_t\) is (short-run) equilibrium private sector employment, PopGrw\(_t\) is the rate of growth of the population 15–64, EcoProKap\(_t\) is the change in the production to capital ratio, ExoTimeFxd\(_t\) is a time trend and ResLabEmpTot\(_t\) is the residual in the labour demand-equation. General government employment, the microeconomic reform index and the rate of growth of population of working age—ExoPubEgg\(_t\), ExoPdyMic\(_t\) and PopGrw\(_t\)—are exogenous.
Appendix C: Labour-market equations in the Murphy Model

This appendix relates the labour-market equations of the Murphy Model as shown in the text above to their representation in the Murphy Model documentation, and provides some further details on the equations.

C.1 The wage-setting equation in the Murphy model

In the text (on page 50), the wage-setting equation in the Murphy Model is shown as:

\[ \Delta \ln(W_t) - \pi_t = c_0 + c_1 \times \Delta UR_{t-1} + c_2 \times \frac{1}{UR_{t-2}} + z_t^W. \]

Expanding the quarter-on-quarter change terms and rearranging gives:

\[ \ln(W_t) = \ln(W_{t-1}) + \lambda_t + c_0 + \pi_t + c_1 \times (UR_{t-1} - UR_{t-2}) + c_2 \times \frac{1}{UR_{t-2}} + z_t^W. \]

The documentation for the Murphy Model works in terms of the employment to labour force ratio, \(E_t/LF_t = 1 - UR_t\). Using this to substitute for the unemployment rate gives:

\[ \ln(W_t) = \ln(W_{t-1}) + \lambda_t + c_0 + \pi_t + c_1 \times \left( \frac{E_{t-2}}{LF_{t-2}} - \frac{E_{t-1}}{LF_{t-1}} \right) + c_2 \times \frac{1}{1 - E_{t-2}/LF_{t-2}} + z_t^W. \]

The corresponding expression in the Murphy Model documentation is:

\[ \ln(W_t) = \ln(W_{t-1}) + AA0SGR_t + C0200 + \left[ \frac{INFE_{t-2}}{800} + \frac{LPCON_{t-1} - LPCON_{t-5}}{8} \right] + C0201 \times \left( \frac{N_{t-2}}{NTS_{t-2}} - \frac{N_{t-1}}{NTS_{t-1}} \right) + C0202 \times \frac{1}{1 - N_{t-2}/NTS_{t-2}} + Z_IW_t, \]

where \(W_t\) is average earnings on a national accounts basis (in $thousand/quarter), AA0SGR_t is the industry-weighted growth in labour efficiency (as a proportion per quarter), \(INFE_t\) is the average expected inflation rate for the next ten years (in per cent per year), \(LPCON_t\) is the log of the constant-utility price index for consumption (on base 1989–90 = 0), \(N_t\) is total employment including defence personnel (in thousands), \(NTS_t\) is labour supply or the labour force (in thousands), and \(Z_IW_t\) is a residual or user-supplied shock. Note that the labour supply variable in the Murphy model is defined to include employment in the defence forces so that the unemployment rate shown in the text is the ratio of unemployment to the total labour force rather than the ratio to the civilian labour force. The term in square brackets on the right-hand side of the equation is expected inflation, \(\pi_t^e\). It is the average of expected average inflation rate over the next ten years and the consumer inflation over the previous year. The expected average inflation rate over the next ten years is given by:

\[ INFE_t = 0.05 \times INF_t + 0.95 \times XPE_t + ZRINFE_t, \]

where \(INF_t\) is the inflation rate for investment goods (in per cent per year), \(XPE_t\) is the model prediction for the inflation rate over the next ten years, and \(ZRINFE_t\) is a user-supplied adjustment (shock) to the equation.
The estimated form of the wage-setting equation is:
\[
\ln(W_t) = \ln(W_{t-1}) + AA0SGR_t - 0.02144295 + \left[ \frac{INFE_{t-2}}{800} + \frac{LPCON_{t-1} - LPCON_{t-5}}{8} \right] \\
- 1.261288 \left( \frac{N_{t-2}}{NTS_{t-2}} - \frac{N_{t-1}}{NTS_{t-1}} \right) + 0.0016 \times \frac{1}{1 - \frac{N_{t-2}}{NTS_{t-2}}} + Z - W_t,
\]

**C.2 Labour supply in the Murphy model**

In the text (on page 53), the short-run labour-supply equation in the Murphy model is shown as:
\[
\ln(LF_t) - \ln(LF^{LR}_{t-1}) = c_0 + c_1 \times t + c_2 \times \ln \left( \frac{LF_{t-1}}{LF^{LR}_{t-1}} \right) + c_3 \times \ln \left( \frac{E_{t-1}}{LF^{LR}_{t-1}} \right) \\
+ c_4 \times \ln \left( \frac{E_{t-2}}{LF^{LR}_{t-2}} \right) + c_5 \times \ln \left( \frac{E_{t-1}}{LF^{LR}_{t-1}} \right) + z_t^N.
\]

If we substitute the definition of the trend labour force as the product of the trend participation rate and the adult population we get:
\[
\ln(LF_t) = \ln(PR^{LR}_t \times POP_{t}^{15+}) + c_0 + c_1 \times t + c_2 \times \ln \left( \frac{LF_{t-1}}{PR^{LR}_{t-1} \times POP_{t}^{15+}} \right) \\
+ c_3 \times \ln \left( \frac{E_{t}}{PR^{LR}_t \times POP_{t}^{15+}} \right) + c_4 \times \ln \left( \frac{E_{t-1}}{PR^{LR}_{t-1} \times POP_{t}^{15+}} \right) \\
+ c_5 \times \ln \left( \frac{E_{t-2}}{PR^{LR}_{t-2} \times POP_{t-2}^{15+}} \right) + z_t^N.
\]

The corresponding expression in the Murphy model documentation is:
\[
\ln(NTS_t) = \ln(PR_t \times [POP_3_t + POP_4_t]) + C0200 + C0201 \times TF_t \\
+ C0202 \times \ln \left( \frac{NTS_{t-1}}{PR_{t-1} \times POP_{t-1} + POP_{t-1}} \right) \\
+ C0203 \times \ln \left( \frac{N_t}{PR_t \times [POP_3_t + POP_4_t]} \right) \\
+ C0204 \times \ln \left( \frac{N_{t-1}}{PR_{t-1} \times [POP_3_{t-1} + POP_4_{t-1}]} \right) \\
+ C0205 \times \ln \left( \frac{N_{t-2}}{PR_{t-2} \times [POP_3_{t-2} + POP_4_{t-2}]} \right) + Z - NTS_t,
\]

where \( NTS_t \) is labour supply or the labour force including defence personnel (in thousands), \( PR_t \) is the underlying trend in the participation rate, \( POP_3 \) is population aged 15–64 (in thousands), \( POP_4 \) is population aged 65 and over (in thousands), \( TF_t \) is a time trend (with base 1989–90 = 0), \( N_t \) is total employment including defence
personnel (in thousands) and $Z_{NTS_t}$ is a residual or user-supplied shock. Note that $PRT_t$, $POP3_t$, $POP4_t$, and $TF_t$ are exogenous.

The estimated form of the labour-supply equation is:

$$
\ln(NTS_t) = \ln(PRT_t \times \left[ POP3_t + POP4_t \right]) - 0.4500523 + 0.0002 \times TF_t
$$

$$
+ 0.7875342 \times \ln \left( \frac{NTS_{t-1}}{PRT_{t-1} \times \left[ POP3_{t-1} + POP4_{t-1} \right]} \right)
$$

$$
+ 0.5023682 \times \ln \left( \frac{N_{t}}{PRT_t \times \left[ POP3_t + POP4_t \right]} \right)
$$

$$
- 0.5793849 \times \ln \left( \frac{N_{t-1}}{PRT_{t-1} \times \left[ POP3_{t-1} + POP4_{t-1} \right]} \right)
$$

$$
+ 0.1896437 \times \ln \left( \frac{N_{t-2}}{PRT_{t-2} \times \left[ POP3_{t-2} + POP4_{t-2} \right]} \right) + Z_{NTS_t}.
$$