

Logic based algebraic cognitive modelling

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Computational Modelling Workshop

Outline

Why algebraic
modelling?

Frith's
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Neuronal firing
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Logic notions

A logic based
algebra: PSEN

Algebraic
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Rationale

- ▶ The abstract model consists of cognitive components with pathways between them.
- ▶ Algebraic expressions can be given for important behaviour in the model.
- ▶ The algebra can be made computable.
- ▶ So we can work towards computer *simulation* of cognitive behaviour:
 - ▶ Abstract from reality to give a *model* of reality.
 - ▶ Postulate principles for model behaviour.
 - ▶ Activate the model according to it's behavioural principles.
 - ▶ Repeatedly analyse and check results against reality:
 - ▶ Modify principles.
 - ▶ Explain and predict reality.

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Frith's examples – 1

- ▶ Schizophrenia is a mental illness. Patients report positive and negative signs and symptoms.
- ▶ A *sign* is an abnormality of behaviour that can be observed.
 - ▶ Negative sign example: *poverty of speech*. If asked a question, answers are restricted to the minimum number of words necessary.
 - ▶ Positive sign example: *incoherence of speech*. Grammar is distorted, there are unexpected shifts of topic, there is a lack of logical connection between sentences.

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Frith's examples – 2

- ▶ A *symptom* is something a patient describes, it cannot be observed.
 - ▶ Positive symptom example: *delusions of control*. Patients experience their actions being controlled by an outside force.
- ▶ Christopher D. Frith gives a (non-algebraic) model that accounts for these in *The Cognitive Neuropsychology of Schizophrenia*, Lawrence Erlbaum Associates 1992.

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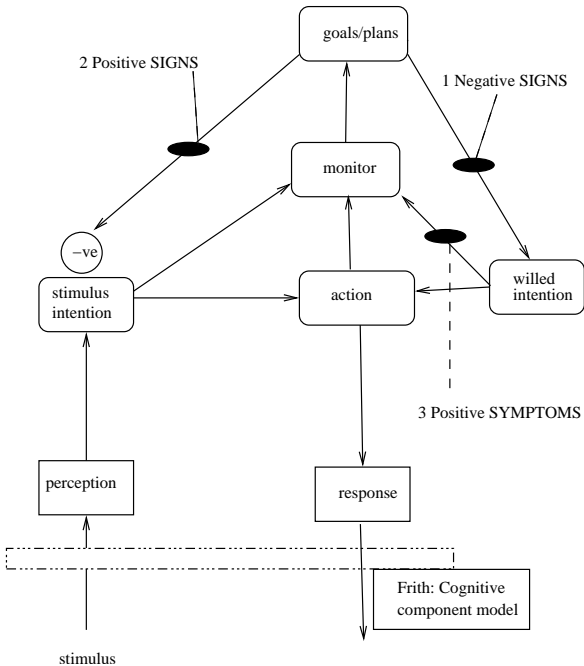
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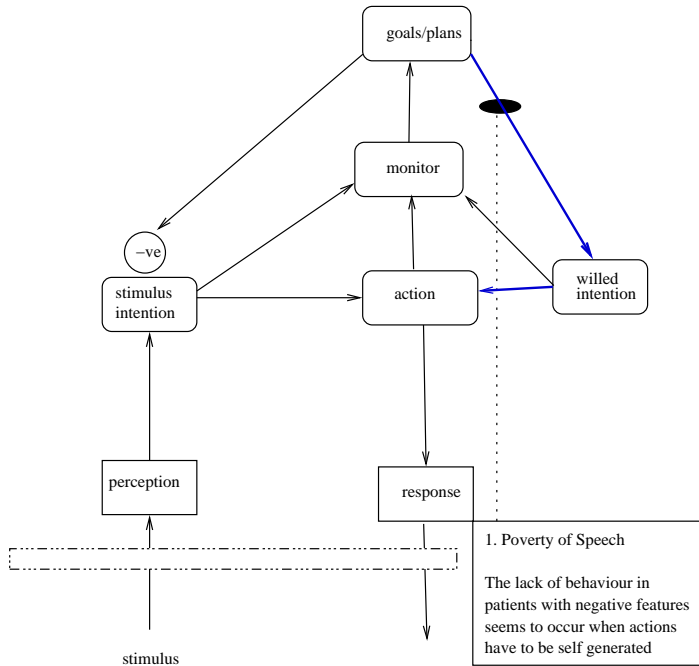
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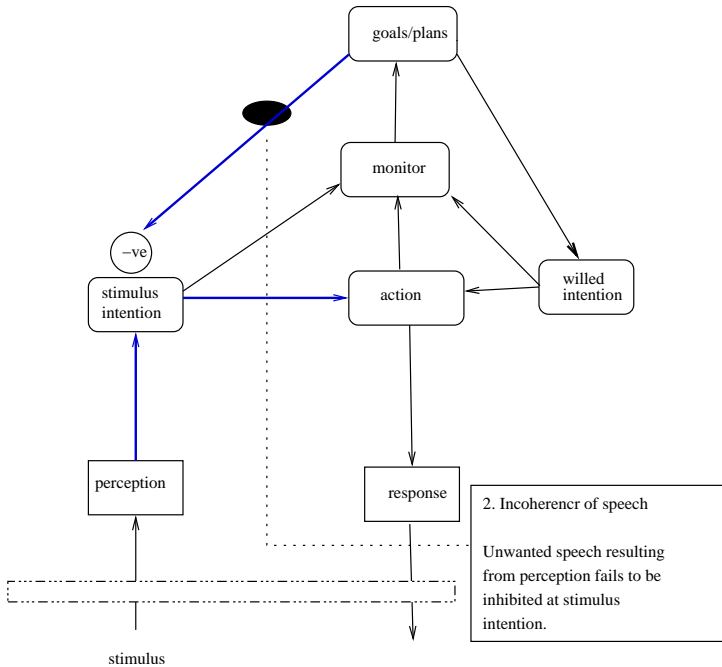
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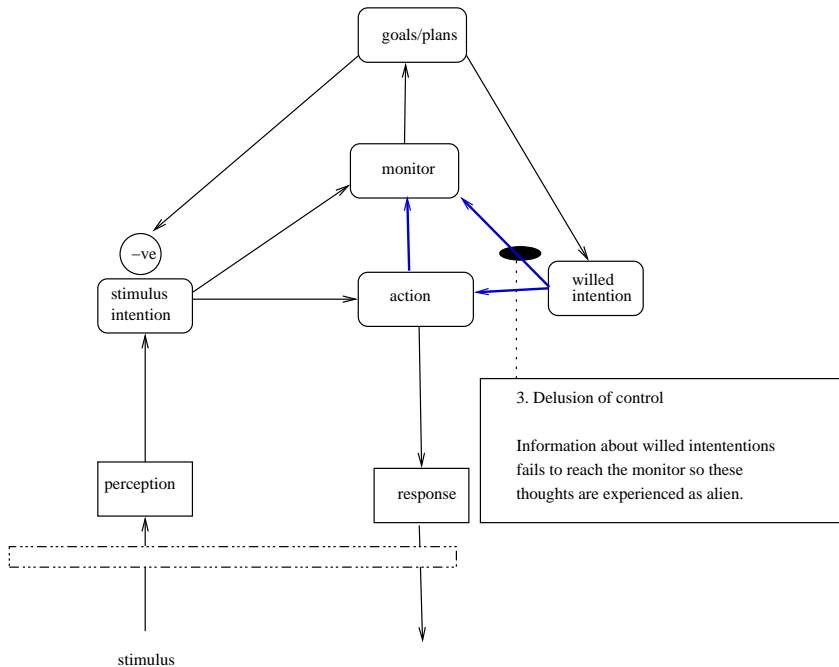
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Firing logic – 1

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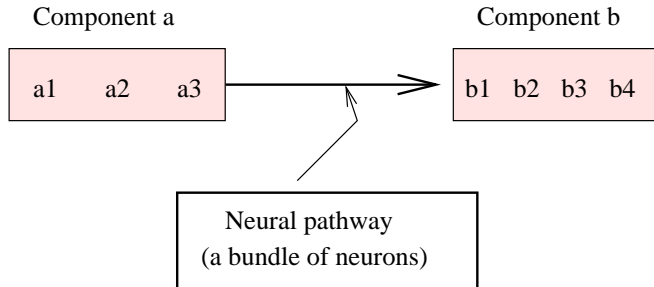
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- ▶ If the b neurons are firing, then write $\text{fires}(\{b1, b2, b3, b4\})$.
- ▶ Or write $\text{fires}(p)$, where $p = \{b1, b2, b3, b4\}$.
- ▶ p is a set of neurons.

Firing logic – 2

- ▶ We have a simple language:
 - ▶ *Constants* – names of sets of neurons, eg. p .
 - ▶ A one place *relation*: **fires**.
 - ▶ *Sentences*: **fires**(p) is a sentence – it has no free variables.
- ▶ When is a sentence true?
- ▶ We need to interpret the sentence in a *structure* and check if the structure *satisfies* the the sentence – then the sentence is *true* in the structure.

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Firing logic – 3

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- ▶ An example of a structure \mathcal{S} : it has
 - ▶ A *domain of interpretation*: a set $\text{dom}(\mathcal{S}) =$ the family of all possible sets of neurons. So $\{\mathbf{a1}, \mathbf{a2}, \mathbf{a3}\}$ and $\{\mathbf{b1}, \mathbf{b2}, \mathbf{b3}, \mathbf{b4}\}$ both belong to $\text{dom}(\mathcal{S})$.
 - ▶ For every name c of a set of neurons, \mathcal{S} maps c to a member of $\text{dom}(\mathcal{S})$, denoted $\mathcal{S}(c)$, the set of neurons named by c . So for example $\mathcal{S}(p) = \{\mathbf{b1}, \mathbf{b2}, \mathbf{b3}, \mathbf{b4}\}$.
 - ▶ \mathcal{S} maps the one place relation symbol, **fires**, to a subset of $\text{dom}(\mathcal{S})$ denoted $\mathcal{S}(\mathbf{fires})$.

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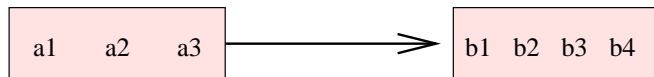
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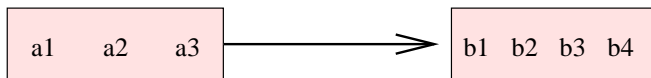
- ▶ $\mathcal{S}(\text{fires})$ is a subset of $\text{dom}(\mathcal{S})$.
- ▶ We expect $\mathcal{S}(\text{fires})$ to reflect reality, so:
- ▶ A set of neurons q belongs to $\mathcal{S}(\text{fires})$ if there
 - ▶ is a pathway ending in a brain component containing q ,
 - ▶ and q fires because some neurons in the source of the pathway fire.
- ▶ So $\{b1, b2, b3, b4\}$ is in $\mathcal{S}(\text{fires})$.



Firing logic – 5

► Summary

- Sentences: $\text{fires}(p)$, $\text{fires}(q)$, $\text{fires}(c)$.
- Interpretation using structures, eg. \mathcal{S} :
 - Domain of interpretation: a set $\text{dom}(\mathcal{S})$.
 - Map constants (or names) to $\text{dom}(\mathcal{S})$, ie. $\mathcal{S}(c)$ is in $\text{dom}(\mathcal{S})$.
 - Map fires to a subset of $\text{dom}(\mathcal{S})$, ie. $\mathcal{S}(\text{fires})$ is a subset of $\text{dom}(\mathcal{S})$.



- When is a sentence true in a structure? Equivalently: when does a structure satisfy a sentence?

Firing logic – 6

- ▶ The sentence $\text{fires}(p)$ is true under \mathcal{S} when $\mathcal{S}(p)$ is in $\mathcal{S}(\text{fires})$.
- ▶ That is when $\{b1, b2, b3, b4\}$ is in $\mathcal{S}(\text{fires})$.
- ▶ But $\mathcal{S}(\text{fires})$ is the family of all sets of neurons that fire when induced by source neuron firing.
- ▶ So $\text{fires}(p)$ is true under \mathcal{S} ; that is \mathcal{S} satisfies $\text{fires}(p)$. This is written $\mathcal{S} \models \text{fires}(p)$.
- ▶ Suppose $q = \{b10, b11\}$ is not induced to fire by any set of neurons at all, then \mathcal{S} does not satisfy $\text{fires}(q)$. This is written $\mathcal{S} \not\models \text{fires}(q)$.
- ▶ The above item is the definition of satisfaction of the negation of $\text{fires}(q)$: $\mathcal{S} \models \neg \text{fires}(q)$ if and only if $\mathcal{S} \not\models \text{fires}(q)$.

A simple language

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- ▶ We have a simple logical language syntax and semantics based on brain components and paths between them:
 - ▶ Names for sets of neurons,
 - ▶ A one place relation, **fires**,
 - ▶ The negation \neg ,
 - ▶ Structures interpreting sentences,
 - ▶ The notion of a structure satisfying a sentence.

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First abstraction step

- ▶ It is difficult to do cognitive modelling with the simple language developed so far: a cognitive modeller cannot easily interpret a firing pattern of neurons.
- ▶ So we take *step 1 in abstraction*: enrich the language to first-order logic.
- ▶ Propose first-order logic as the “language of thought” in our modelling.

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Language of Thought

- ▶ The “Language of Thought” theory
 - ▶ J. A. Fodor, *The Language of Thought*, 1976,
 - ▶ Aydede, Murat, *The Language of Thought Hypothesis*, The Stanford Encyclopedia of Philosophy (Fall 2004 edition), Edward N. Zalta (ed.), <http://plato.stanford.edu/archives/fall2004/entries/language-thought/>
- ▶ says:
 - ▶ Human thinking is based on *concepts* and that these are governed by syntax and semantics so that
 - ▶ Complex thoughts are built from simple ones, and the semantics (meaning) of a complex thought can be understood by combining the semantics of its simple constituents.
- ▶ Our approach is to model the language of thought using a first-order (logical) language.

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First-order logic

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- ▶ We have
 - ▶ A countable number of *constants* (names),
 - ▶ A countable number of *function symbols*,
 - ▶ A countable number of *relation symbols*,
 - ▶ A countable number of *individual variables*,
 - ▶ Connectives \neg (not), \wedge (and), \vee (or), \Rightarrow (implies),
 - ▶ Quantifiers: \forall (for all), \exists (there exists).
- ▶ Sentence example: $(\forall x)\text{man}(x) \Rightarrow \text{mortal}(x)$.
- ▶ Structures and satisfaction of *sentences*.

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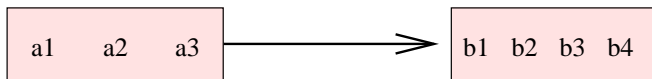
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Second abstraction step



$$A \vDash_R B$$

- ▶ Neuronal firing pattern at a brain component *abstracts to* a set of first-order sentences.
- ▶ Neuron pathway R *abstracts to* restricted entailment, \vDash_R .
- ▶ Restricted entailment:
 - ▶ R is a set of structures.
 - ▶ For a set of sentences X , mod_X consists of those structures lying in R that satisfy X :
 $\text{mod}_R X = R \cap \{\mathcal{S} : \mathcal{S} \vDash X\}$.
 - ▶ $X \vDash_R Y$ if and only if $\text{mod}_R X \subseteq \text{mod}_R Y$.

Modelling Frith's Account – 1

- ▶ Each cognitive component (*perception, response, action, monitor, etc.*) has a set of sentences associated with it (Language of Thought).
- ▶ Let A and B be cognitive components with a pathway from one to the other thus: $A \rightarrow B$.
- ▶ We propose that at any point in time there is an active set of sentences at each cognitive component, and that for a pathway such as $A \rightarrow B$ A 's active sentences induce an active set at B .

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Modelling Frith's Account – 2

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- ▶ The basic idea: there is a boolean algebra, PSEN, whose operations can combine sets of sentences at a component to give meaningful results.
- ▶ Recall the two element boolean algebra $(0, 1, <, \wedge, \vee)$. It has an order $<$, bounds with respect to $<$, and an inverse:
 - ▶ LUB: $0 \vee 1$ is 1, etc.
 - ▶ GLB: $0 \wedge 1$ is 0, etc.
 - ▶ INVERSE: -0 is 1, etc.

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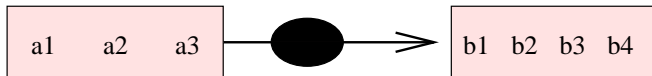
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- ▶ The elements of the algebra PSEN are sets of sentences.
- ▶ Take \models to be the order relation of the algebra.
- ▶ *Note that, to make the notation less cluttered, the subscript R has been dropped throughout; it is understood to be there implicitly.*
- ▶ GLB $\{X, Y\}$ is $X \wedge Y$ and it satisfies
 - ▶ $\text{mod}(X \wedge Y) \sim \text{mod}(X) \cap \text{mod}(Y)$ and
 - ▶ $X \wedge Y \sim \text{Cn}X \cup \text{Cn}Y$.
- ▶ LUB $\{X, Y\}$ is $X \vee Y$ and it satisfies
 - ▶ $\text{mod}(X \vee Y) \sim \text{mod}(X) \cup \text{mod}(Y)$ and
 - ▶ $X \vee Y \sim \text{Cn}X \cap \text{Cn}Y$.

- ▶ PSEN has
 - ▶ Bottom, \perp , where $\text{mod}(\perp) = \emptyset$.
 - ▶ Top, \top_R , where $\text{mod}_R(\top) = R$.
- ▶ The inverse of X , written $-X$, exists and satisfies $X \wedge -X \sim \perp$.
- ▶ Bottom behaves like 0, Top like 1: $X \wedge \perp \sim \perp$, $X \vee \perp \sim X$, $X \wedge \top \sim X$ and $X \vee \top \sim \top$.
- ▶ There is also a *contraction* operator:
 - ▶ For sets of sentences X and Y , we *contract* X by Y when we remove from X (in a minimal way) any sentences that will entail Y .
- ▶ The operations of PSEN are computable under certain (reasonable) conditions.

Modelling a disconnection

$$A \vDash_R B$$



- ▶ Model a disconnection by taking $R = \emptyset$.
- ▶ Now for *any* X and Y , $X \vDash_{\emptyset} Y$ because $\text{mod}_{\emptyset} X = \emptyset = \text{mod}_{\emptyset} Y$ and $X \vDash_{\emptyset} Y$ iff $\text{mod}_{\emptyset} X \subseteq \text{mod}_{\emptyset} Y$.
- ▶ Note also, $\text{mod}_{\emptyset} \top = \emptyset$.
- ▶ So if there is a disconnection make a canonical choice and take Y to be \top .

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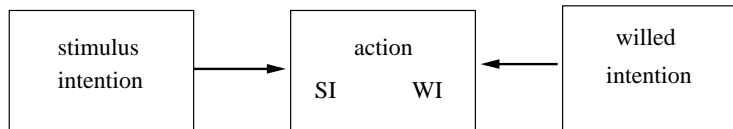
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- ▶ Cognitive modelling will be done algebraically by computing expressions corresponding to:
 - ▶ *Combination* to explain Poverty of Speech,
 - ▶ *Contraction* to explain Incoherence of Speech,
 - ▶ *Comparison* to explain Delusion of Control.

Combination explains Poverty of Speech

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- ▶ Execution plan for **action**: $WI \wedge SI$.
- ▶ $WI \wedge SI \sim Cn(WI) \cup Cn(SI)$.

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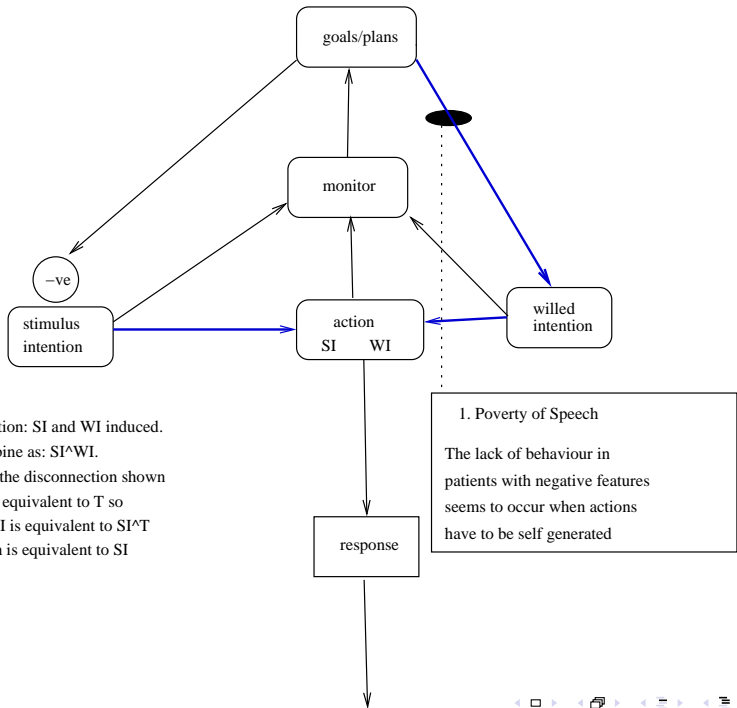
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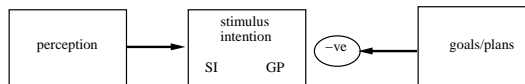
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At action: SI and WI induced.
 Combine as: $SI \wedge WI$.
 With the disconnection shown
 WI is equivalent to T so
 $SI \wedge WI$ is equivalent to $SI \wedge T$
 which is equivalent to SI

1. Poverty of Speech
 The lack of behaviour in patients with negative features seems to occur when actions have to be self generated

Contraction Models Inhibition



- ▶ Model inhibition using *contraction*.
- ▶ All sentences in *SI* that contradict, or entail contradiction of *GP* must be removed. This is called *contraction*.
- ▶ Contradiction of *GP* is $\neg GP$, so we need to contract *SI* by $\neg GP$. We need to *reject* $\neg GP$.
- ▶ Denote the rejector of X by M_X . We require $M_X \models \neg X$.
- ▶ We need $M_{\neg GP} \models \neg(\neg GP) \sim GP$.
- ▶ The contraction formula is $SI \vee M_{\neg GP}$; this can be shown not to entail $\neg GP$.

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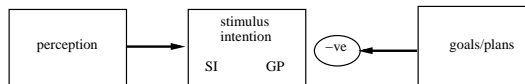
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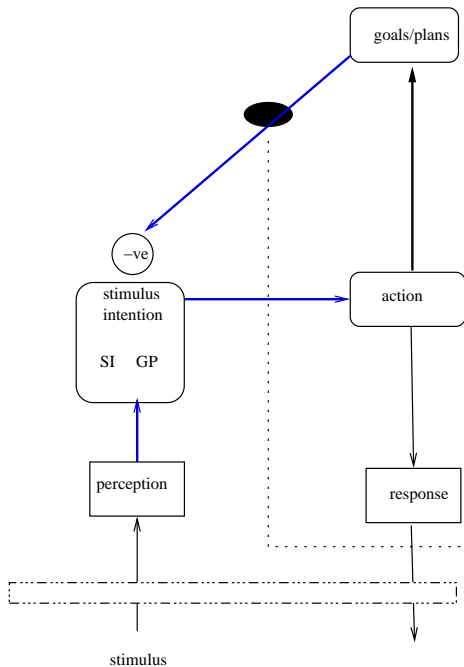
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Contraction Example



- ▶ $SI = \{macU(Lee), teaRoom(Lee)\}$,
 $GP = \{workshopSession(Lee)\}$.
- ▶ Suppose $\neg teaRoom(Lee) \models workshopSession(Lee)$.
- ▶ We know that
 $teaRoom(Lee) \not\models workshopSession(Lee)$.
- ▶ As a rejector take $M_{GP} = \{\neg teaRoom(Lee)\}$, then
 $M_{GP} = \neg teaRoom(Lee) \models workshopSession(Lee) \sim GP$.
- ▶ Note that we need $M_{GP} \models -(-GP) \sim GP$.
- ▶ The contraction is
 $(SI \vee M_{GP}) \sim macU(Lee) \vee \neg teaRoom(Lee)$ and
this does not entail $-GP$.



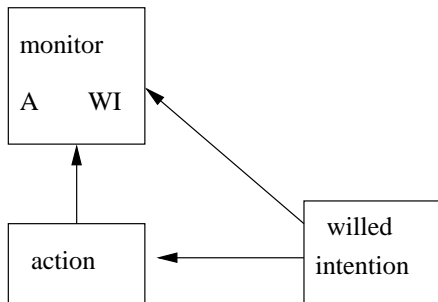
At stimulus intention: SI and GP are induced.

Normal operation (no disconnection) inhibits non GP by contracting by -GP. This is SIvM.

With disconnection 2, GP is T and so -GP is bottom; then so is M. So SIvM is SI and the contraction is SI. It follows that inappropriate speech fails to be inhibited.

2. Incoherencr of speech
 Unwanted speech resulting from perception fails to be inhibited at stimulus intention.

Comparison explains Delusions of Control



- ▶ Monitor compares A and WI for equivalence:
- ▶ If $(WI \vee A) - (WI \wedge A) \sim \perp$ (bottom), then $WI \sim A$.
- ▶ Equivalence lets monitor conclude that action plans are sourced from within.

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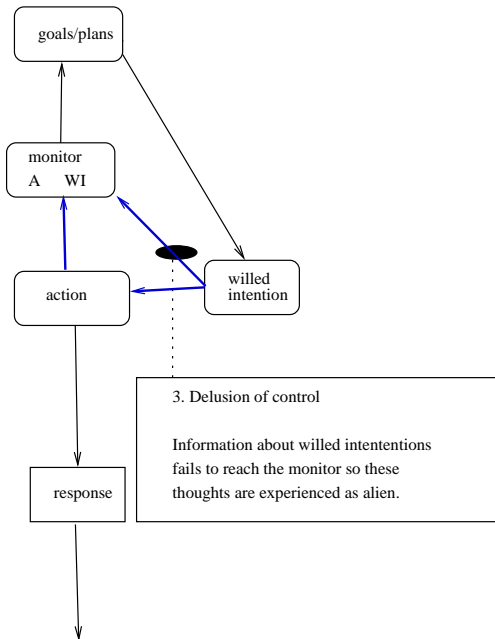
At monitor, WI and A induced

Compare them to see if they are the same or different. Use the symmetric difference and check that it is different from bottom.

Compute $(WI \vee A) - (WI \wedge A)$.

With the disconnection, WI is T and the difference is $(T \vee A) - (T \wedge A)$ which is equivalent to $(T - A)$, which is equivalent to $\neg A$.

This is different from bottom.



Theory of Mind Mechanism – Tomm

- ▶ A theory of mind mechanism (tomm) explains how one is able to understand what is going on in another mind.
- ▶ Leslie has given a model of tomm.
 - ▶ Alan M. Leslie, *Pretense and representation: the origins of “theory of mind”*, 1987.
- ▶ We consider the activity of pretence as an example of tomm. Suppose we are pretending to use a banana as a telephone.
- ▶ Leslie postulates the existence of a specific cognitive component called a *decoupler* to handle processing of the pretence. See next slide.
- ▶ Significantly, Gallagher and Frith suggest that the site of the cognitive component that processes decoupling is the *anterior paracingulate cortex*. This is located centrally towards the front of the head.

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Neuronal firing
logic

Logic notions

A logic based
algebra: PSEN

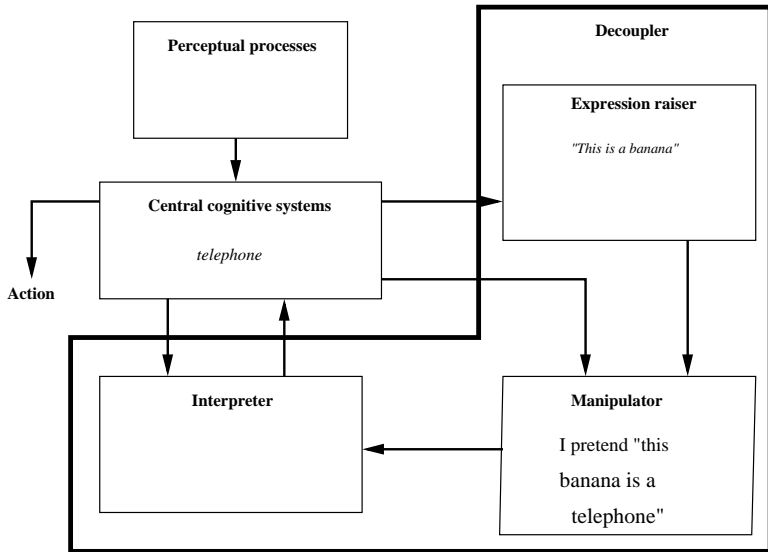
Algebraic
modelling of
Frith's
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Theory of Mind
Mechanism –
Tomm

Tomm example

Conclusion

Leslie's decoupler model of pretence



Tomm example

- ▶ Pretend play scenario.
 - ▶ A cup is held above a surface and we pretend that the cup holds liquid. When the cup is upturned we pretend that the surface is wet (even though it is not really wet).
- ▶ We use *first-order modal logic* to formalise the scenario and reason with its elements. This coherent with our logic-based account of Frith's examples given earlier.
 - ▶ The *necessity modality*, [Nec], is used to indicate that certain sentences apply everywhere. For example, "It is necessary that if a cup containing liquid is upturned above a surface, then the surface will be wet".
 - ▶ The *pretence modality*, <Pret>, is used to indicate that a certain sentence is a pretence. For example, "Pretend the cup contains liquid".
- ▶ We use *semantics* to decide whether a sentence is true or false.

Modal semantics

- ▶ We use Kripke possible world semantics.
- ▶ There are, in general, many possible worlds.
- ▶ The truth (or falsity) of a modal sentence has to be considered at a world; truth cannot be assessed in isolation without reference to a world.
- ▶ As part of the semantic setup we are told, for each world, which other worlds it can “see” or *access*. This is used in the assessment of the truth of a sentence.
 - ▶ The sentence $[\text{Nec}]Y$ is true at world w if Y (without the $[\text{Nec}]$ prefix) is true at each world accessible from w .
 - ▶ The sentence $\langle \text{Pret} \rangle Y$ is true at world w if Y (without the $\langle \text{Pret} \rangle$ prefix) is true at some (at least one) world accessible from w .

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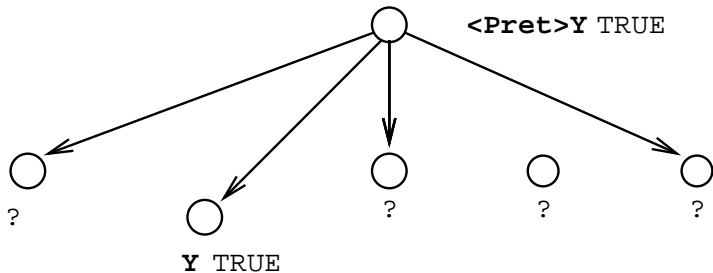
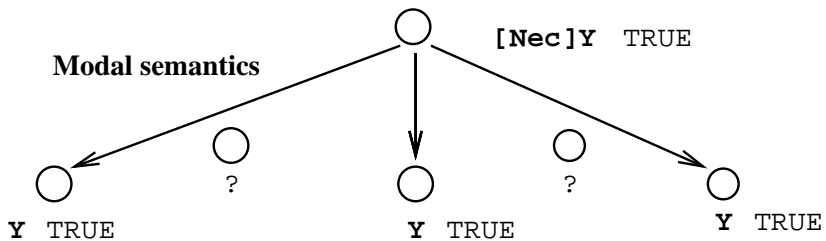
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Tomm example again

- ▶ Scenario: A cup is held above a surface and we pretend that the cup holds liquid. When the cup is upturned we pretend that the surface is wet (even though it is not really wet).
- ▶ Using logical argument can we pretend that the surface is wet?

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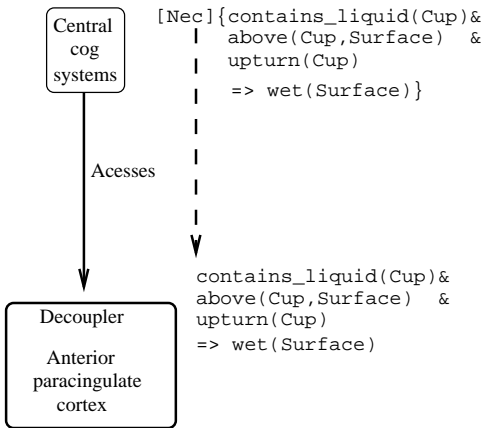
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Central
cog
systems

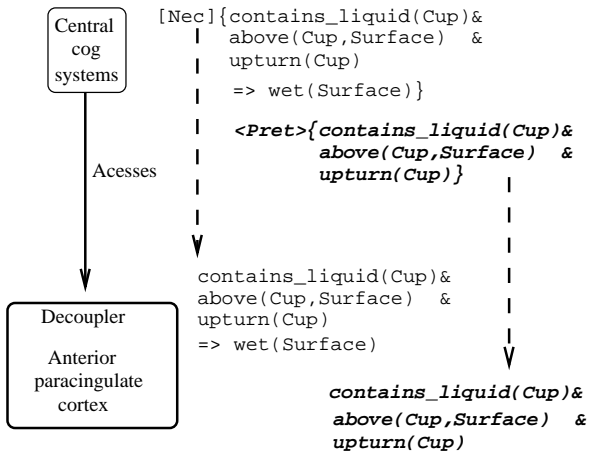
```
[Nec]{contains_liquid(Cup)&  
      above(Cup, Surface) &  
      upturn(Cup)  
      => wet(Surface)}
```

Accesses

Decoupler
Anterior
paracingulate
cortex



--- ➤ denotes entailment



- - - - - > denotes entailment

Central
cog
systems

Accesses

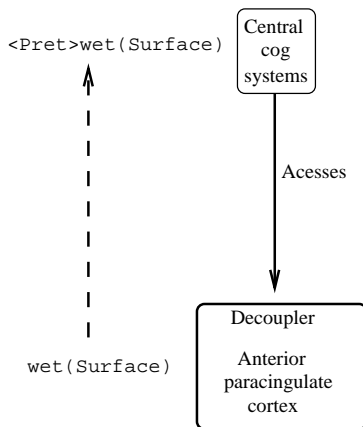
Decoupler
Anterior
paracingulate
cortex

`contains_liquid(Cup) &
above(Cup, Surface) &
upturn(Cup)
=> wet(Surface)`

`wet(Surface)`

**`contains_liquid(Cup) &
above(Cup, Surface) &
upturn(Cup)`**

---> denotes entailment



---> denotes entailment

<Pret>wet(Surface)

Central
cog
systems

[Nec]{contains_liquid(Cup)&
above(Cup, Surface) &
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Accesses

<Pret>{contains_liquid(Cup)&
above(Cup, Surface) &
upturn(Cup)}

wet(Surface)

Decoupler
Anterior
paracingulate
cortex

contains_liquid(Cup)&
above(Cup, Surface) &
upturn(Cup)
=> wet(Surface)

**contains_liquid(Cup)&
above(Cup, Surface) &
upturn(Cup)**

---> denotes entailment

Summary

- ▶ An account of human cognitive activity has been given with the following features:
 - ▶ It is an abstraction from the physical architecture of brain components and neural pathways.
 - ▶ Brain components are modelled as cognitive components.
 - ▶ Each cognitive component has a set of logical sentences associated it.
 - ▶ The sentences are in first-order modal logic – abstracted as the “language of thought”.
 - ▶ A neural pathway from X to Y is modelled as an entailment from sentences of X , SX , to those of Y , SY .
 - ▶ A logic-based algebra can be used to combine sets of sentences induced by different entailments targeting the same cognitive component, giving an algebra of cognitive processing.
 - ▶ Modal semantics can be used to model the pretence aspect of Leslie’s decoupler in the tomm.

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Future Work

- ▶ Work needs to be done to model the following:
 - ▶ The working of *memory* as a cognitive process.
 - ▶ Conditions where tomm seems not to operate normally such as *autism*.
- ▶ In another direction work on cognitive modelling as outlined here would benefit from having a system that allowed simulation. A computer system based on the kind of algebra used here would allow this.

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